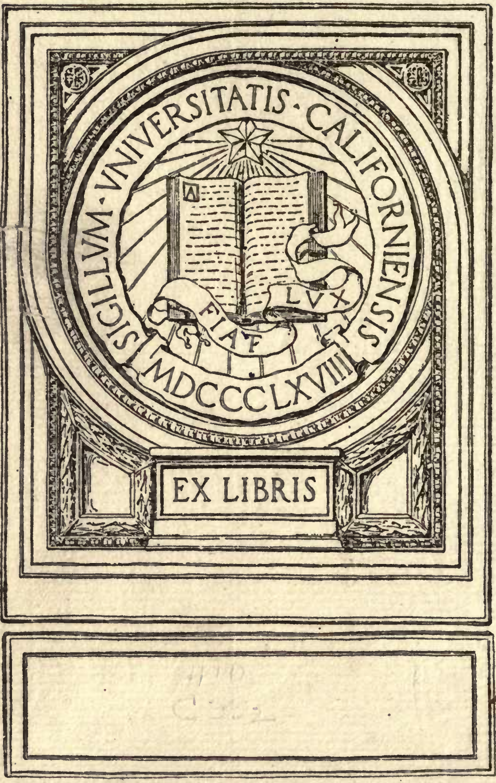
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SOLUTION OF RAILROAD PROBLEMS BY THE SLIDE RULE

BY

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PREFACE

The ease and rapidity of solving problems in Railroad Curves by the use of the slide rule led the author to develop a set of problems for class-room work. The object of this book is to present similar problems for the convenience of students who have studied Railroad Curves and the Theory of the Slide Rule.

Where it is possible the solution of the problems should be made by the use of the *C* and *D* scales of the Mannheim slide rule on account of their greater precision.

The notation used in Allen's "Railroad Curves and Earthwork" is used thruout this discussion.

The discussion of the slide rule is from Professor C. W. Crockett's article in the *Polytechnic* of May 2, 1910.

The discussion of the slide rule, the development of the equations used and the discussion of the easement curve have been added to make this book of more general interest.

E. R. Cary.

Troy, N. Y.
March 1, 1913.

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THE SOLUTION
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BY
THE SLIDE RULE

THIS discussion will be given under the following heads: The Slide Rule, Problems in Simple Curves, Problems in Compound Curves, The Vertical Curve, Turnouts, The Easement Curve, and Problems in Earthwork.

CHAPTER I

THE SLIDE RULE

1. The Mannheim and Carpenter rules are the most common types of the slide rule. The face of either bears two kinds of scales, on one of which, for the ordinary ten-inch rule, the distance from 1 to 1 is about 5 inches, while on the other the distance from 1 to 1 is about 10 inches, the latter distance being exactly double the former.

Let us call the shorter scale the single scale and the longer scale the double scale; on the

Mannheim rule, the single scale is called *A* or *B* and the double scale is *C* or *D*; on the Carpenter rule, the single scale is called *A* or *B*, and the double scale is called *C*. Note that the double scale is the long scale.

Let a and b be constant numbers and x a third number to which different values are to be assigned, and suppose we wish to find the values of y in any one of the following expressions:

$$y = \frac{ax}{b},\ y = \frac{ax^2}{b^2},\ y = \sqrt{\frac{ax}{b}},\ y = \sqrt{\frac{ax^2}{b^2}},$$

$$y = \frac{ab}{x},\ y = \frac{ab^2}{x^2},\ y = \sqrt{\frac{ab}{x}},\ y = \sqrt{\frac{ab^2}{x^2}},$$

or of any similar expression containing two factors in the numerator and one in the denominator, any one or more being squared, a being a constant factor in the numerator.

The method of setting the instrument is as follows:

1°. If x is in the numerator, use the slide direct.

If x is in the denominator, use the slide inverted.

2°. The numbers a, b, x and y are found on the instrument in the order named; a on

the rule, b on the slide, x on the slide, y on the rule; that is, we look on the rule and slide in the following order: *Rule, slide, slide, rule.* Or we could find a on the slide, b on the rule, x on the rule, y on the slide; that is, we look on the rule and slide in this order, *Slide, rule, rule, slide.*

3°. If a, b or x is raised to the first power, we must find it on a single scale; if squared, on a double scale.

4°. If y is a first power, we must find it on a single scale; if a square root, on a double scale.

Thus to find the value of $y = \frac{ax^2}{b^2}$ with the Mannheim rule, we may proceed in two ways, using the slide direct, since x is in the numerator:

(a) Find a on the *single* scale on the *rule,* and opposite it set b on the *double* scale on the *slide;* find x on the *double* scale on the *slide,* and opposite it read the result y on the *single* scale on the *rule.* Or:

(b) Find a on the *single* scale on the *slide,* and opposite it set b, found on the *double* scale on the rule; find x on the *double* scale on the rule, and opposite it read the result y on the *single* scale on the *slide.*

The same problem may be solved with the Carpenter or Thacher rule, but only by the second method, since the slide does not bear a double scale; this limitation is not objectionable, however, except in continued operations, where it is desirable to use the runner.

Had we wished to find the value of $y = \sqrt{\frac{ax^2}{b^2}}$, the settings on the Mannheim rule would have been the same as before, but the result y would have been on the *double* scale instead of on the single scale. The Carpenter or Thacher rule cannot solve this problem without first reading on the slide the value of $\frac{ax^2}{b^2}$, and then determining its square root.

2. The Mannheim slide rule is used in the solution of the problems given herein. The scales of this rule are as follows: The upper scale of the rule, A; the upper scale of the slide, B; the lower scale of the slide, C; the lower scale of the rule, D. On the back of the slide are the sine and tangent scales, and when these are to be used the slide must be reversed, i.e., the slide changed in the rule so that the back face is brought to the front.

CHAPTER II

SIMPLE CURVES

3. The center line of a railroad track consists of tangents and curves (circular arcs), and, in modern practice, also of some form of easement curve connecting them.

4. In this country a curve is designated by its degree. The degree of a curve is the central angle subtended by a chord of one hundred feet. If the metric system is used, the degree of a curve may be defined as the central angle subtended by a chord of ten meters. In almost all other countries a curve is designated by its radius.

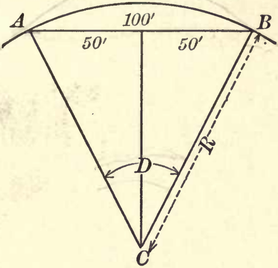


Fig. 1

From Fig. 1,

$$R \sin \tfrac{1}{2} D = 50;$$

$$R = \frac{50}{\sin \frac{1}{2} D}.$$

As D is usually a small angle,

$$R = \frac{50}{\frac{1}{2} D \sin 1'},$$

or

$$R = \frac{100 \dfrac{1}{\sin 1'}}{D}.$$

$$\frac{1}{\sin 1'} = 3438;$$

$$R = \frac{100 \times 3438}{D}. \quad . \quad . \quad . \quad (1)$$

In Equation (1), D is expressed in minutes and R in feet. For metric curves R in meters $= \dfrac{10 \times 3438}{D'}$.

Problem 1. Find the radius of a $6° 30'$ curve. To solve by the slide rule, opposite 3438 on the D scale set D in minutes on the C scale, opposite the index of the C scale read the result on the D scale for R in feet.

By inverting the slide and setting 3438 on the C scale opposite the index of the D scale, the radius of any degree of curvature may be found on the D scale opposite the degree of the curve in minutes on the inverted C scale.

For the above problem the setting is as follows:

Opposite 3438 on the D scale set 390 on the

C scale and opposite the right index of the C scale read 882 on the D scale, or $R = 882$ feet. Except for odd minute curves the approximate formula, $R = \frac{5730}{D}$, gives an easy solution by the slide rule. In this formula D is in degrees. If this formula is figured exactly and $\frac{7}{100}$ of D is added to the value found for R, the result is practically a precise value of R.

5. To find the decimal point, always figure the result roughly by mental arithmetic, e.g., in the above problem use 3900 in place of 3438 and the result is 1000, showing that the correct result is about 1000. Hence, when the figures 882 are obtained by the slide rule, the decimal point is placed after the third figure for the value of R.

6. The functions of a curve are: T, the tangent distance; E, the external distance; M, the middle ordinate; C, the long chord; P.C., the beginning of the curve; P.T., the ending of the curve; and I the central angle of the curve.

In Fig. 2,
$$T = AV = VB.$$
$$E = VF.$$
$$M = FG.$$
$$C = AB.$$
$$I = ACB = BVX.$$

From Fig. 2 the following formulas are readily derived:

$$T = R \tan \tfrac{1}{2} I. \quad \ldots \ldots \quad (2)$$

$$E = R\,(\sec \tfrac{1}{2} I - 1) = R \operatorname{exsec} \tfrac{1}{2} I. \quad (3)$$

$$\operatorname{exsec} \tfrac{1}{2} I = \tan \tfrac{1}{2} I \tan \tfrac{1}{4} I,$$

or $$E = R \tan \tfrac{1}{2} I \tan \tfrac{1}{4} I \quad \ldots \quad (4)$$

$$M = R - R \cos \tfrac{1}{2} I = R \operatorname{vers} \tfrac{1}{2} I \quad . \quad (5)$$

$$\operatorname{vers} \tfrac{1}{2} I = 2 \sin^2 \tfrac{1}{4} I = \sin \tfrac{1}{2} I \tan \tfrac{1}{4} I,$$

or $$M = R \sin \tfrac{1}{2} I \tan \tfrac{1}{4} I \quad \ldots \quad (6)$$

$$C = 2 R \sin \tfrac{1}{2} I. \quad \ldots \ldots \quad (7)$$

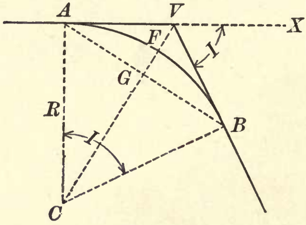


Fig. 2

Problem 2. Find the tangent distance for a 6° 12′ curve with a central angle of 30° 24′.

$T = R \tan \frac{1}{2} I = R \tan 15° 12'$.

R, found as in Problem 1, = 925 feet.

$T = 925 \tan 15° 12' = 251.5$ feet.

The setting for T is as follows: Using the slide reversed, opposite 925 on the D scale set the right index of the T scale and opposite $15° 12'$ on the T scale read 251.5 on the D scale. In figuring roughly for the decimal point use the sin or tan of $1° = 0.0175$ and assume that the tangents and sines vary as the angles, i.e., the sin of 15° is 15 times the sin of 1°. This is only approximately true for angles less than 30°.

Problem 3. Find the external distance for a $6° 12'$ curve with a central angle of $30° 24'$.

$$E = R \tan \tfrac{1}{2} I, \tan \tfrac{1}{4} I$$
$$= R \tan 15° 12', \tan 7° 36'.$$

R, the same as in Problem 2, is 925 feet.

$E = 925 \tan 15° 12', \tan 7° 36' = 33.5$ feet.

The setting for E is as follows: Using the slide reversed, set the right index of the T scale opposite 925 on the D scale, bring the runner to $15° 12'$ on the T scale, then bring the left index of the T scale under the runner, and opposite $7° 36'$ on the T scale read the result 33.5 on the D scale. The position of the decimal point is found as indicated in Problem 2.

Problem 4. Find the middle ordinate for a $6° 12'$ curve with a central angle of $30° 24'$.

$M = R \sin \frac{1}{2} I \tan \frac{1}{4} I.$

R, the same as in Problem 2, is 925 feet.

$M = 925 \sin 15° 12' \tan 7° 36' = 32.4$ feet.

The setting for M is as follows: Using the slide reversed, opposite 925 on the D scale, set the right index of the T scale and opposite $7° 36'$ on the T scale read 1235 on the D scale. Opposite 1235 on the A scale set the right index of the S scale and opposite $15° 12'$ on the S scale read 32.4 on the A scale. The position of the decimal point is found as indicated in Problem 2.

Problem 5. Find the length of the long chord of a $6° 12'$ curve with a central angle of $30° 24'$.

R, found as above, $= 925$ feet.

$C = 2 R \sin \dfrac{I}{2} = 2 R \sin 15° 12' = 2 \times 242 =$ 484 feet.

Setting for $925 \sin 15° 12'$ is similar to that given in Problem 4.

7. In bending rails it is convenient to use the middle ordinate for a given length of chord to determine the degree of the curve to which the rail is bent. The middle ordinate

may be expressed in terms of the length of the chord and the radius or in terms of the length of the chord and the degree of the curve. These formulas are derived as follows:

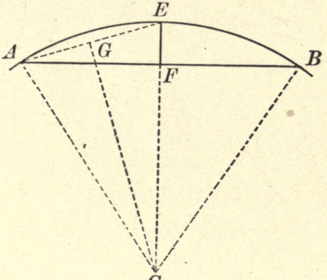


Fig. 3

In Fig. 3, AB is the chord, c, and EF is its middle ordinate. G is at the middle of AE.

$$AE = AF, \text{ approximately.}$$

Then $$\frac{AE}{EF} = \frac{AC}{AG}$$

or $$\frac{\frac{1}{2}c}{M} = \frac{R}{\frac{1}{4}c} \quad \text{or} \quad M = \frac{c^2}{8R}. \quad . \quad . \quad . \quad (8)$$

$$R = \frac{5730}{D}.$$

$$M = \frac{c^2D}{8 \times 5730} = \frac{c^2D}{45{,}840}.$$

Problem 6. Find the middle ordinate of a chord 84 feet long on a 6° 12′ curve.

$$M = \frac{c^2D}{45,840} = 84^2 \times \frac{6.2}{45,840} = 0.953 \text{ foot.}$$

The setting for M is as follows: Opposite 84 on the D scale set 4584 on the right B scale, opposite 62 on the left B scale read 953 on the A scale. Figure roughly for the decimal point.

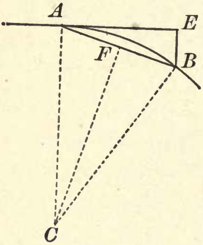


Fig. 4

8. In Fig. 4, EB is the tangent offset, a, and its value in terms of chord, c, and the radius is found as follows:

Triangles AEB and ACF are similar, hence

$$\frac{EB}{AB} = \frac{AF}{AC},$$

but $$AC = R,\ AF = \tfrac{1}{2}\,c.$$

Then $$\frac{a}{c} = \frac{\frac{1}{2}\,c}{R} \quad \text{or} \quad a = \frac{c^2}{2\,R}. \quad . \quad . \quad . \quad . \qquad (9)$$

Problem 7. Find the tangent offset for a 6° 12′ curve for a chord of 110 feet.

$$a = \frac{c^2}{2\,R}.$$

R, found as above, $= 925$ feet.

$$a = \frac{110^2}{2 \times 925} = \frac{0.5 \times 110^2}{925} = 6.55 \text{ feet.}$$

Setting for A is: Opposite 5 on the A scale set 925 on the B scale and opposite 11 on the C scale read 655 on the A scale.

9. The tangent offset may be used to lay out a curve by the use of a tape only. In Fig. 5, if A is at station $16 + 40$, then

$$\overline{A\,17} = 60 \text{ feet, and } \overline{E\,17} = \frac{60^2}{2R} = \frac{60^2 D}{11{,}460}.$$

$$\overline{A\,17} - \overline{AE} = \frac{\overline{E\,17}^2}{2\,\overline{A\,17}}.$$

From this $\overline{AE}$ may be found.

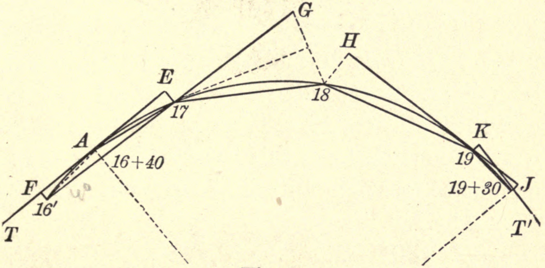


Fig. 5

It is assumed that the direction of the line TA is established on the ground and that A

is located. Then AE may be measured out and E located. Then measuring off $\overline{A\,17}$ from A and $\overline{E\,17}$ from E, their intersection may be found and 17 located as a point on the curve. Point 16′ may be located in a manner similar to locating 17. Both 16′ and 17 are on the curve. By prolonging $\overline{16', 17}$, 100 feet from 17 to G, measuring $\overline{G\,18} = 2\,a_{100}$ (where a_{100} is the tangent offset for a chord of 100 feet), and measuring 100 feet from 17, the intersection of these measurements locates station 18. In a similar manner station 19 may be located from the prolongation of $\overline{17, 18}$.

$$\overline{18\,H} \text{ is } a_{100}. \quad \overline{18, 19} - \overline{19\,H} = \frac{{a_{100}}^2}{200}.$$

From this $\overline{19\,H}$ may be found. Then by measuring $\overline{18\,H}$ from 18 and $\overline{19\,H}$ from 19, H may be located and the line $\overline{19\,H}$ is tangent to the curve at station 19. Then station 19 + 30 may be located from this tangent in a manner similar to locating station 17 from the tangent thru A.

$\overline{19\,K} = \dfrac{30^2}{2\,R}$. 30 minus the distance from 19 + 30 to K equals $\dfrac{\overline{19\,K}^2}{60}$. From these dis-

tances K may be located, and K joined with station 19 + 30 gives the direction of the tangent ahead of the curve.

10. A curve may be located by offsets from a long chord. The method of finding the offsets is as follows:

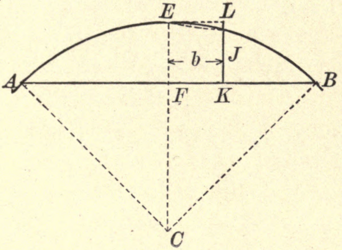


Fig. 6

In Fig. 6, $JK = LK - LJ.$

$LK = EF.$

Let $AB = c$, and $FK = b$.

Then JK, the offset at K, $= \dfrac{c^2}{8R} - \dfrac{b^2}{2R}$,

or $$JK = \frac{\left(\frac{c}{2}\right)^2}{2R} - \frac{b^2}{2R} = \frac{\left(\frac{c}{2}\right)^2 - b^2}{2R}$$

$$= \frac{\left(\frac{c}{2} + b\right)\left(\frac{c}{2} - b\right)}{2R} \quad . \quad . \quad . \quad . \quad (10)$$

By Equation (10) the offset to the curve from any point on the chord is equal to the product of the segments of the chord divided by twice the radius of the curve.

Problem 8. Find the offset to a 6° 12′ curve from a point 24 feet from the middle of a chord 104 feet long.

$$O = \frac{\left(\frac{c}{2} + b\right)\left(\frac{c}{2} - b\right)}{2\,R}.$$

R, found as above, = 925 feet.

$$O = \frac{76 \times 28}{2 \times 925} = \frac{76 \times 28}{1850} = 1.15 \text{ feet.}$$

Setting for O is: Opposite 76 on the D scale set 185 on the C scale; shift the slide to the left thru its entire length by bringing the runner over the left index of the C scale and running the slide thru until the right index of the C scale is under the runner, and opposite 28 on the C scale read 115 on the D scale.

11. Curves are usually laid out by deflection angles. The deflection angle is the angle between a tangent at a given point and a chord thru the same point. The deflection angles ordinarily used for laying out a curve are those between the tangent thru the be-

ginning of the curve and the chords from the beginning of the curve to the stations on the curve and to the end of the curve.

The angle between two chords is measured by one-half of the arc intercepted by them. If this arc is subtended by a chord 100 feet long, the angle between the two chords is $\frac{1}{2} D$ (D being the degree of the curve). If the arc is subtended by any other length of chord, its value is found as follows:

Let c be the chord subtending the arc and d be the angle at the center subtended by the arc. Then (as already shown by Fig. 2), $c = 2R \sin \frac{1}{2} d$ or $\sin \frac{1}{2} d = \frac{c}{2R}$. $\operatorname{Sin} \frac{1}{2} D = \frac{100}{2R}$. Then $\frac{\sin \frac{1}{2} d}{\sin \frac{1}{2} D} = \frac{c}{100}$. Usually angles d and D are small and for small angles the sines are proportional to the angles.

$$\frac{\sin \frac{1}{2} d}{\sin \frac{1}{2} D} = \frac{d}{D} = \frac{c}{100} \text{ or } d = \frac{c}{100} D.$$

The application of this is shown in the following problem.

Problem 9. Find the deflections for a 6° 12′ curve with a central angle of 45° 36′ and its P.C. at station 106 + 42. The length of any

curve may be found in stations by dividing the central angle by the degree of the curve.

Then

$$L = \frac{45° \, 36'}{6° \, 12'} = \frac{2736}{372} = 7.355 \text{ stations.}$$

$$\begin{array}{lcr} \text{P.C. at} & & 106 + 42 \\ L & = & 7 + 35.5 \\ \hline \text{P.T. at} & & 113 + 77.5 \end{array}$$

Let d_1 be the deflection for station 107 and d_2 the deflection for the last chord.

$$d_1 = \frac{372 \times 58}{200} = 108'$$

(58 feet from 106 + 42 to 107).

$$d_2 = \frac{372 \times 77.5}{200} = 144'$$

(77.5 feet from 113 to P.T.).

Station.	Deflection.
106 + 42 P.C.	....
107	1° 48′
108	4° 54′
109	8° 00′
110	11° 06′
111	14° 12′
112	17° 18′
113	20° 24′
113 + 77.5 P.T.	22° 48′

Setting for d_1 and d_2 is: Opposite 372 of the D scale set 2 on the C scale and opposite 58 and 77.5 on the C scale read 108 and 144 respectively on the D scale.

The deflection for station 107 is $108' = 1° 48'$. The deflection for station 108 is found by adding $3° 06' (\frac{1}{2} D°)$ to the deflection for station 107. The deflection for station 109 is found by adding $3° 06'$ to that for station 108, and so on, until the deflection for station 113 is found. The deflection for station 113 + 77.5 is found by adding $144' = 2° 24'$ to the deflection for station 113. As the deflection for station 113 + 77.5 is one-half of the central angle of the curve, the computation checks.

12. Another method of locating a curve is by offsets from the tangent thru any given point of the curve. The method is shown in Problem 10.

Problem 10. Find the necessary distances to locate a $6° 12'$ curve from station 91 + 40 to station 94, by offsets from the tangent thru station 91 + 40.

$$< BA\,\overline{92} = \frac{60}{100}\,\frac{6° 12'}{2} = \frac{60 \times 186'}{100} = 111.5'$$
$$= 1° 51.5'.$$

$$< C'\,\overline{92}\,\overline{93} = 2 \times 1° 51.5' + \frac{6° 12'}{2} = 6° 49'.$$

$$< D'\,\overline{93}\,\overline{94} = 6°\,49' + 6°\,12' = 13°\,1'.$$

$$\overline{B\,92} = 60 \sin 1°\,51.5' = 1.86 \text{ feet.}$$

$$\overline{C'\,93} = 100 \sin 6°\,49' = 11.85 \text{ feet.}$$

$$\overline{D'\,94} = 100 \sin 13°\,1' = 22.50 \text{ feet.}$$

$$\overline{C\,93} = \overline{B\,92} + \overline{C'\,93} = 13.71 \text{ feet.}$$

$$\overline{D\,94} = \overline{C\,93} + \overline{D'\,94} = 36.21 \text{ feet.}$$

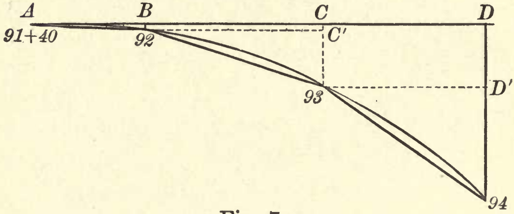


Fig. 7

In Fig. 7,

$$\overline{A\,92} - AB = \frac{\overline{B\,92}^2}{2 \times \overline{A\,92}} = \frac{1.86^2}{120} = 0.03,$$

or $$AB = 60 - 0.03 = 59.97 \text{ feet.}$$

$$\overline{92\,C'} = \overline{92\,93} - \frac{\overline{C'\,93}^2}{2 \times \overline{92\,93}} = 100 - \frac{11.85^2}{200}$$

$$= 100 - 0.7, \text{ or } BC = 99.30 \text{ feet.}$$

$$\overline{93\,D'} = \overline{93\,94} - \frac{\overline{D'\,94}^2}{2 \times \overline{93\,94}} = 100 - \frac{22.50^2}{200}$$

$$= 100 - 2.53, \text{ or } CD = 97.47 \text{ feet.}$$

$AC =$ . . . . 159.27 feet.

$AD =$ . . . . 256.74 feet.

The settings are similar to those given in Problems 5 and 7.

13. It is sometimes desired to change a curve so that it may end in a parallel tangent nearer to or further from the center than the tangent in which it ends. Three cases arise in practice, as follows:

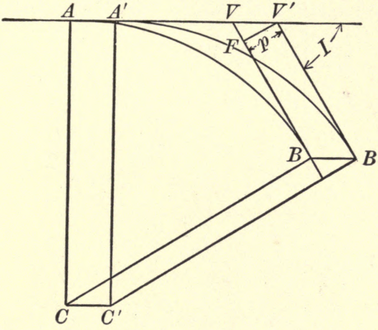


Fig. 8

1st. By using same degree of curve, find new P.C. and P.T.

$$AA' = VV' = BB' = \frac{p}{\sin I}.$$

Add AA' to station of old P.C. for the station of the new P.C. and add AA' to station of the old P.T. for the station of the new P.T.

2nd. By keeping same P.C., find new degree of curve and new P.T.

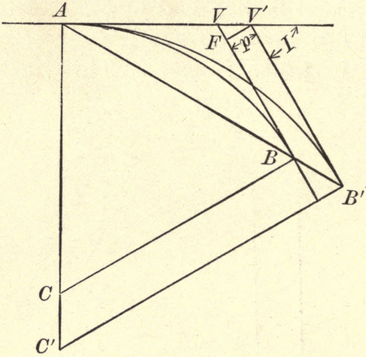


Fig. 9

In Fig. 9,

$$VV' = \frac{p}{\sin I}.$$

$$AV' = AV + VV'.$$

$$AV = R \tan \frac{I}{2}.$$

$$AV' = R' \tan \frac{I}{2}.$$

$$R' \tan\frac{I}{2} = R \tan\frac{I}{2} + \frac{p}{\sin I}.$$

$$R' = R + \frac{p}{\tan\frac{I}{2}\sin I}.$$

3rd. By ending the curve directly opposite old P.T., find new degree of curve, new P.C. and new P.T.

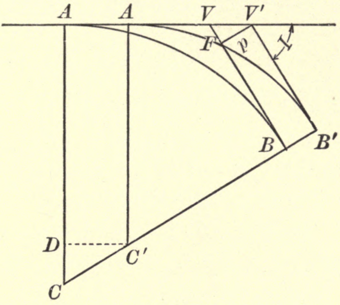


Fig. 10

In Fig. 10,

$$VB = V'B' + VF.$$

$$VB = R \tan \tfrac{1}{2} I.$$

$$V'B' = R' \tan \tfrac{1}{2} I.$$

$$VF = \frac{p}{\tan I}.$$

Then

$$R \tan \tfrac{1}{2} I = R' \tan \tfrac{1}{2} I + \frac{p}{\tan I} \quad . \quad . \quad (11)$$

$$R' = R - \frac{p}{\tan \frac{1}{2} I \tan I}.$$

$$AA' = DC' = (R - R') \tan I.$$

From Equation (11),

$$(R - R') \tan I = \frac{p}{\tan \frac{1}{2} I}.$$

Then $$AA' = \frac{p}{\tan \frac{1}{2} I}.$$

Add to the station of the old P.C. $\overline{AA'}$ for the station of the new P.C. From R' the new degree of the curve can be found. From the new degree of the curve and the central angle the length of the new curve can be found, and from the new station of the P.C. and the length of the new curve the new station of the P.T. can be found.

These equations are better adapted to the slide-rule solutions than the ordinary ones for these problems. If the new ending tangent is nearer the center, a change in the signs will result in the above equations.

Problem 11. Find the stations of the new P.C. and P.T. for ending a 3° 30′ curve in a

parallel tangent 13 feet further from the center, keeping the degree of the curve the same. The central angle is 22° 24′ and old P.C. is at station 126 + 40.

$$AA' = \frac{13}{\sin 22^\circ\, 24'} = 34.1 \text{ feet.}$$

Setting: Opposite 13 on the *A* scale set 22° 24′ on the *S* scale of the slide reversed and opposite the right index of the *S* scale read 34.1 on the *A* scale.

$$L = \frac{22^\circ\, 24'}{3^\circ\, 30'} = \frac{1344}{210} = 6.40 \text{ stations.}$$

Old P.C. at station	126 + 40
	34.1
New P.C. at station	126 + 74.1
$L =$	6 + 40
New P.T. at station	133 + 14.1

Problem 12. Using the same curve as in Problem 11, it is desired to move ending tangent 13 feet nearer the center, keeping the same P.C.

$$R' = R - \frac{p}{\tan\frac{I}{2}\sin I}.$$

$$R = \frac{100 \times 3438}{210} = 1635 \text{ feet.}$$

$$\frac{p}{\tan \frac{1}{2} I \sin I} = \frac{13}{\tan 11^\circ 12' \sin 22^\circ 24'} = 172.1.$$

$$R' = 1635 - 172.1 = 1462.9 \text{ feet.}$$

Setting for $\dfrac{13}{\tan 11^\circ 12' \sin 22^\circ 24'}$ is: Opposite 13 on the A scale set 22° 24′ on the S scale of the slide reversed and opposite the right index of the S scale read 341 on the A scale; opposite 341 on the D scale set 11° 12′ on the T scale and opposite the left index of the T scale read 172.1 on the D scale.

$$D' = \frac{100 \times 3438}{1463} = 234' = 3^\circ 54'.$$

$$L' = \frac{22^\circ 24'}{3^\circ 54'} = \frac{1344'}{234'} = \quad 5 + 75.$$

$$\begin{array}{ll} \text{P.C. at} & \underline{126 + 40} \\ \text{P.T. at} & 132 + 15 \end{array}$$

Problem 13. Using the same curve as in No. 11 and ending new curve directly opposite old P.T. in a new parallel tangent 13 feet further from the center, find new degree of curve and new stations of the P.C. and the P.T.

$$R' = R - \frac{p}{\tan \frac{1}{2} I \tan I} = 1635$$

$$- \frac{13}{\tan 11^\circ \, 12' \tan 22^\circ \, 24'} = 1635 - 159.$$

$R' = 1476$ feet.

Setting for $\dfrac{13}{\tan 11^\circ \, 12' \tan 22^\circ \, 24'}$ is: Opposite 13 on the D scale set $11^\circ \, 12'$ on the T scale of the slide reversed, bring the runner to the right index of the T scale, bring $22^\circ \, 24'$ on the T scale under the runner and opposite the left index of the T scale read 159 on the D scale.

$$D' = \frac{100 \times 3438}{1476} = 233' = 3^\circ \, 53'.$$

$$L' = \frac{22^\circ \, 24'}{3^\circ \, 53'} = \frac{1344}{233} = 5.78 \text{ stations.}$$

$$AA' = (R - R') \tan \tfrac{1}{2} I + \frac{p}{\sin I}$$

$$= 159 \tan 11^\circ \, 12' + \frac{13}{\sin 22^\circ \, 24'}$$

$$= 31.5 + 34.1 = 65.6 \text{ feet.}$$

Old P.C. at	126 + 40
	65.6
New P.C. at	127 + 5.6
L' =	5 + 78
New P.T. at	132 + 83.6

14. Other special problems in simple curves may be solved with fair precision by the use of the slide rule. To get good results it is necessary that the angles whose sines are used shall be between 0° and 70°, and those whose cosines are used shall be between 20° and 90°. The following is a sample of such special problems.

Problem 14. Find the degree of the simple curve that will pass thru a given point and will join two tangents having a given intersection angle. The values are given in Fig. 11.

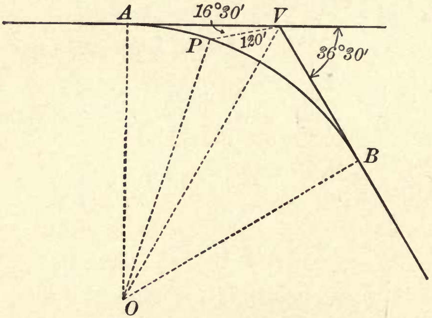


Fig. 11

In Fig. 11,

$$OVP = \frac{180° - 36°\ 30'}{2} - 16°\ 30'$$

$$= 90° - 34°\ 45' = 55°\ 15'.$$

$$OP = R.$$

$$OV = \frac{R}{\cos 18^\circ 15'}.$$

In the triangle OVP,

$$\frac{R}{\dfrac{R}{\cos 18^\circ 15'}} = \frac{\sin 55^\circ 15'}{\sin OPV}.$$

$$\sin OPV = \frac{\sin 55^\circ 15'}{\cos 18^\circ 15'} = \frac{\sin 55^\circ 15'}{\sin 72^\circ 45'}.$$

Solving this by the slide rule,

$$\sin OPV = \sin 60^\circ.$$

The angle OPV is greater than 90°, as seen by the construction of Fig. 11. Hence

$$OPV = 180^\circ - 60^\circ = 120^\circ.$$

The setting of the slide rule for finding the sin OPV is as follows:

Reverse the slide in the rule. Make the indices of the A scale and the sine scale to coincide, move the runner over 55° 15′ on the sine scale, bring 71° 45′ on the sine scale under the runner, move the runner to the right index of the sine scale and then set the indices of the sine scale to agree with the indices of the A scale, reading the result 60° under the runner.

$$POV = 180° - (120° + 55° 15') = 4° 45'.$$

$$R = 120 \frac{\sin 55° 15'}{\sin 4° 45'}.$$

This solved by the slide rule gives

$$R = 1190 \text{ feet.}$$

$$D, \text{ in minutes, } = \frac{100 \times 3438}{R} = 289' = 4° 49'.$$

It is probable that a 5° curve would be used under the conditions given.

15. In the remaining problems the settings of the slide rule will not be given, except in a few cases, as those already given show the methods that usually may be applied.

CHAPTER III

Compound Curves

16. It is sometimes necessary, for the best location, to combine curves of different radii. When these curves are on the same side of their common tangent, they form a *compound*

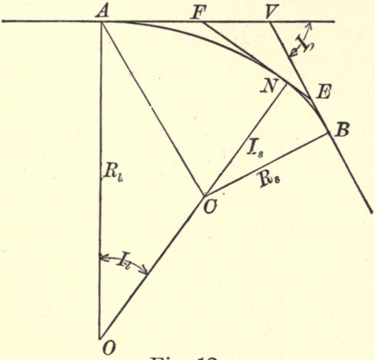


Fig. 12

curve and when on opposite sides of their common tangent, they form a *reversed curve.* A reversed curve may have branches of equal or unequal radii. In a compound curve the radii of the branches are always unequal.

Fig. 12 shows a compound curve, the tangent distances, T_l and T_s, for which may be found as follows:

$$T_l = AV. \qquad T_s = VB.$$
$$R_l = AO. \qquad R_s = CB.$$
$$FE = FN + NE = R_l \tan \tfrac{1}{2} I_l + R_s \tan \tfrac{1}{2} I_s.$$

FV and *VE* may be found by solving the triangle *FVE*.

Then

$$T_l = R_l \tan \tfrac{1}{2} I_l + FV.$$
$$T_s = R_s \tan \tfrac{1}{2} I_s + VE.$$

Problem 15. Find the tangent distances for the compound curve consisting of a 3° curve with a central angle of 14° 20′ beginning at *A* and compounding with a 6° curve with a central angle of 40° 40′ ending at *B*.

$$R_l = \frac{100 \times 3438}{180} = 1910 \text{ feet.}$$
$$R_s = \frac{100 \times 3438}{360} = 955 \text{ feet.}$$
$$FE = 1910 \tan 7^\circ\, 10' + 955 \tan 20^\circ\, 20'$$
$$= 240 + 354 = 594.$$
$$FV = \frac{594}{\sin 55^\circ} \sin 40^\circ\, 40' = 473.$$

$$VE = \frac{594}{\sin 55^\circ} \sin 14^\circ 20' = 179.5.$$

$$T_l = 240 + 473 = 713 \text{ feet.}$$

$$T_s = 354 + 179.5 = 533.5 \text{ feet.}$$

17. Nearly all problems in compound curves may be solved by the method of coördinates, the application of which is given in the following three problems.

Problem 16. The deflection angle of the tangents of a compound curve is 52°. The curve begins at station 372 and the first branch is a 6° curve to the left, 5 stations long. The T_s is to be 550 feet. Find the radius and the degree of the second branch and the stations of the P.C.C. and the P.T.

In Fig. 12, the P.C. of the curve, to fulfil the condition given in the problem, is at B.

$$I_s = 6 \times 5 = 30^\circ.$$

$$I_l = I - I_s = 52^\circ - 30^\circ = 22^\circ.$$

$$R_s = \frac{5730}{6} = 955'.$$

The lengths and directions of the lines NC, CB and BV are known, and, using NC as a meridian, their L's and M's are found as follows:

Line.	Azimuth.	Length.	L		M	
			+	−	+	−
		Feet.				
NC.......	0° 00′	955	955	.....	.....	.....
CB.......	210° 00′	955	.....	827	.....	477
BV.......	120° 00′	550	.....	275	476	.....
			955	1102	476	477
VN.......	....	....	147	.....	1	.....

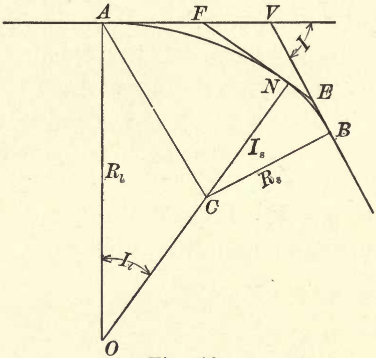


Fig. 12

$$l_2 \cos 210° = -955 \cos 30° = -827.$$
$$(\cos 30° = \sin 60°.)$$
$$l_2 \sin 210° = -955 \sin 30° = -477.$$
$$l_3 \cos 120° = 550 \cos 60° = 275.$$
$$(\cos 60° = \sin 30°.)$$

$l_3 \sin 120° = 550 \sin 60° = 476.$

Tan of azimuth of $VN = \frac{1}{147} = 0.0068,$

or azimuth of $VN = 23'$.

$$\text{Length of } VN = 147 + \frac{1^2}{2 \times 147} = 147.03,$$

practically 147 feet.

In Fig. 12, the azimuth of VA is $120° - 52° = 68°$. The azimuth of VN is $0° \ 23'$ and of NA is the angle $ANO = \frac{1}{2}(180° - 22°) =$

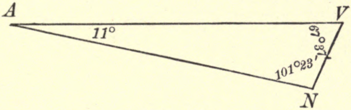


Fig. 13

79°. To find the lines AV and NA, the triangle shown in Fig. 13 is solved:

$$AV = \frac{147}{\sin 11°} \sin 101° \ 23' = 755 \text{ feet.}$$

$$NA = \frac{147}{\sin 11°} \sin 67° \ 37' = 711 \text{ feet.}$$

$$R_l = \frac{NA}{2 \sin \frac{1}{2} I_l} = \frac{711}{2 \sin 11°} = 1865 \text{ feet.}$$

$$D_l = \frac{3438}{1865} \times 100 = 184.5' = 3° \ 04.5'.$$

In practice a 3° curve would probably be used.

$$L_l = \frac{22°}{3°} = \qquad 7 + 33.3$$

P.C.C. is at $\quad 377$

P.T. is at $\quad \overline{384 + 33.3}$

$T_l = AV = 755$ feet.

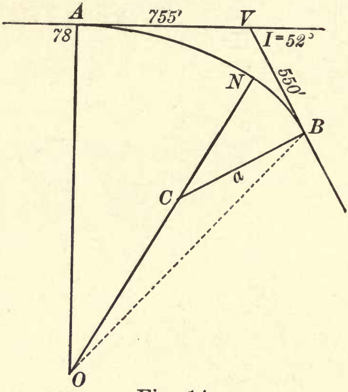


Fig. 14

Problem 17. Given the data shown in Fig. 14, find the degree of the second branch of the compound curve and the stations of the P.C.C. and the P.T., beginning the compound curve with a 3° (the long radius branch) curve at station 78.

Use OA as a meridian.

Line.	Azimuth.	Length.	L +	L −	M +	M −
		Feet.				
OA.......	0° 00′	1910	1910	.....	.....	.....
AV.......	90° 00′	755	.....	.....	755	
VB.......	142° 00′	550	.....	434	338	.....
			1910	434	1093	.....
BO.......	....	....	.....	1476	.....	1093

$L_3 = 550 \cos 142° = - 550 \sin 52° = - 434.$
$M_3 = 550 \sin 142° = 550 \sin 38° = 338.$

For the line OB the coördinates would be positive.

Tan of the azimuth of $OB = \frac{1093}{1476} = 0.741.$

Az. of $OB = 36° 30'.$

Az. of $BO = 216° 30'.$

Az. of $BC = 142° + 90° = 232°.$

Angle $a = 232° - 216° 30' = 15° 30'.$

In the triangle CBO, $BC = R_s$, $CO = R_l - R_s$,

$$OB = \frac{1093}{\sin 36° 30'} = 1838 \text{ feet.}$$

$$s = \frac{1838 + R_l - R_s + R_s}{2} = 919 + \frac{1910}{2} = 1874.$$

$$\operatorname{Sin}^2 \tfrac{1}{2} a = \frac{(s - R_s)(s - 1838)}{1838 R_s} = \frac{(1874 - R_s)\,36}{1838\,R_s}$$

$$= \frac{1874 - R_s}{51\,R_s}.$$

$\operatorname{Sin}^2 \frac{1}{2}(15° 30') = \sin^2 7° 45' = 0.01815.$

The setting for $\sin^2$ 7° 45′ is as follows: Reverse the slide in the rule, set the indices of the sine scale and the *A* scale coincident and read on the *A* scale 1348 opposite 7° 45′ on the sine scale. Then set the runner over 1348 on the *D* scale and read 1815 on the *A* scale under the runner.

$$0.01815 \times 51 \times R_s = 1874 - R_s,$$

or $$R_s = 974 \text{ feet}.$$

To find 51 × 0.01815, keep the runner in the position stated above, bring the numbered face of the slide to the front, set the left index of the *B* scale under the runner and read 925 on the *A* scale opposite 51 on the *B* scale.

$$D_s = \frac{3438 \times 100}{974} = 353' = 5° 53'.$$

Use a 6° curve.

$$\operatorname{Sin} I_s = \frac{1838}{1910 - 974} \sin 15° 30'.$$

$$I_s = 31° 40'.$$

$$I_l = 52° - 31° 40' = 20° 20'.$$

$$L_l = \frac{20.3333}{3} = 6 + 77.8 \text{ stations.}$$

$$L_s = \frac{31.6666}{6} = 5 + 27.7 \text{ stations.}$$

P.C.	at	86 + 00
L_l	=	6 + 77.8
P.C.C.	at	92 + 77.8
L_s	=	5 + 27.7
P.T.	at	98 + 05.5

Problem 18. Begin a compound curve with a 4° (long radius) curve at station 378. Find the degree of the second branch and stations of

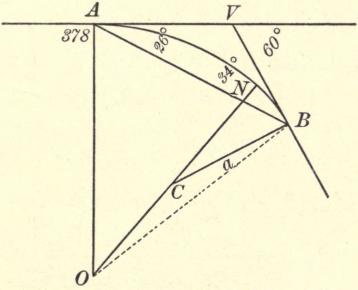


Fig. 15

P.C.C. and P.T. of the compound curve to connect A and B under the conditions given in Fig. 15.

Line.	Azimuth.	Length.	L +	L −	M +	M −
		Feet				
OA......	0° 00′	1432.5	1432.5	.....	.....	.....
AB......	116° 00′	1200	......	525	1079	.....
BO......	....	....	......	907.5	.....	1079

Using *OA* as a meridian.

$$1200 \cos 116° = -525.$$

$$1200 \sin 116° = 1079.$$

$$\text{Tan azimuth } OB = \frac{1079}{907.5},$$

or

$$\text{azimuth } OB = 49° \; 56'.$$

$$\text{Azimuth } BC = 240° \; 00'$$

$$\text{Azimuth } BO = 229° \; 56'$$

$$a = \overline{10° \; 04'}$$

$$BO = \frac{1079}{\sin 49° \; 56'} = 1408.*$$

$$s = \tfrac{1}{2}(1432.5 - R_s + R_s + 1408) = 1420.25.$$

$$\text{Sin}^2 \tfrac{1}{2} a = \frac{(s - R_s)(s - BO)}{R_s \overline{BO}}$$

$$= \frac{(1420.25 - R_s)\, 12.25}{1408 \, R_s}.$$

* To get a good result care must be taken to read this value on the slide rule very carefully.

$$0.0077 = \frac{(1420.25 - R_s)\,12.25}{1408\,R_s},$$

or

$$R_s = 755 \text{ feet.}$$

$$D_s = \frac{3438 \times 100}{755} = 455' = 7^\circ\,35'.$$

Use a $7^\circ\,30'$ curve.

$$R_l - R_s = 1432.5 - 755 = 677.5.$$

$$\text{Sin } COB = \frac{755}{677.5} \sin 10^\circ\,04',$$

or

$$COB = 11^\circ\,12'.$$

$$I_s = 10^\circ\,04' + 11^\circ\,12' = 21^\circ\,16'.$$

$$I_l = 60^\circ - 21^\circ\,16' = 38^\circ\,44'.$$

$$L_l = \frac{38^\circ\,44'}{4^\circ} = 9 + 68.3.$$

$$L_s = \frac{21^\circ\,16'}{7^\circ\,30'} = 2 + 83.5.$$

P.C.	at	378
L_l	=	9 + 68.3
P.C.C.	at	387 + 68.3
L_s	=	2 + 83.5
P.T.	at	390 + 51.8

Problem 19. A 4° curve begins at station 395 + 30 and ends at station 408 + 40. Find the station of the P.C.C., where a 6° curve

will compound so as to end in a parallel tangent 12 feet nearer the center.

$$EK = CO = R_l - R_s = EB.$$
$$BB' = (R_l - R_s) \text{ vers. } I_s$$
$$= (R_l - R_s)(1 - \cos I_s).$$
$$R_l - R_s = 1432.5 - 955 = 477.5.$$
$$\text{Cos } I_s = 1 - \frac{12}{477.5} = 0.9749.$$

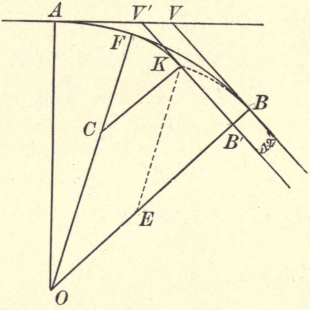


Fig. 16

$$\text{Sin}\,(90 - I_s) = 0.9749, \quad \text{or} \quad 90 - I_s = 77°.*$$
$$I_s = 13°.$$
$$I = 13.10 \times 4 = 52°\ 24'.$$
$$I_l = 52°\ 24' - 13° = 39°\ 24'.$$

* Reading for this must be carefully made.

$$L_s = \frac{13°}{6°} = 2 + 16.7.$$

$$L_l = \frac{39°\ 24'}{4°} = 9 + 85.$$

$$\begin{array}{lcr} \text{P.C.} & \text{at} & 395 + 30 \\ \text{P.C.C.} & \text{at} & \overline{405 + 15} \\ L_s & = & 2 + 16.7 \\ \text{P.T.} & \text{at} & \overline{407 + 31.7} \end{array}$$

Problem 20. An 8° curve begins at station 372 and ends at station 378 + 50. Find the stations of the P.C.C.'s and the P.T. for a three-centered compound curve, with one station of a 1° curve at each end and passing thru the same P.C. and P.T. as the 8° curve.

$$\frac{O_2O_3}{O_1O_3} = \frac{\sin \frac{1}{2} I}{\sin \frac{1}{2} I_s} = \frac{R_l - R_s}{R_l - R},$$

$$I = 6.5 \times 8 = 52°.$$

$$I_s = 52° - 2° = 50°.$$

$$R_l - R = 5729.7 - 716.8 = 5012.9.$$

$$R_l - R_s = 5012.9 \frac{\sin 26°}{\sin 25°} = 5200.$$

$$R_s = 5729.7 - 5200 = 529.7 \text{ feet.}$$

$$D_s = \frac{3438 \times 100}{529.7} = 650' = 10°\ 50'.$$

$$L_s = \frac{50°}{10° 50'} = 4 + 61.$$

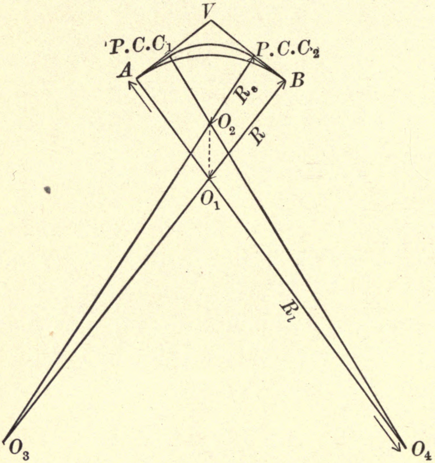


Fig. 17

P.C.	at	372 + 00
L_l	=	1 + 00
P.C.C.$_1$	at	373 + 00
L_s	=	4 + 61
P.C.C.$_2$	at	377 + 61
L_l	=	1 + 00
P.T.	at	378 + 61

Problem 21. Given two straight parallel tracks with center lines 13 feet apart and with a long chord of 186.4 feet, find the degree of the equal branches of a reversed curve to con-

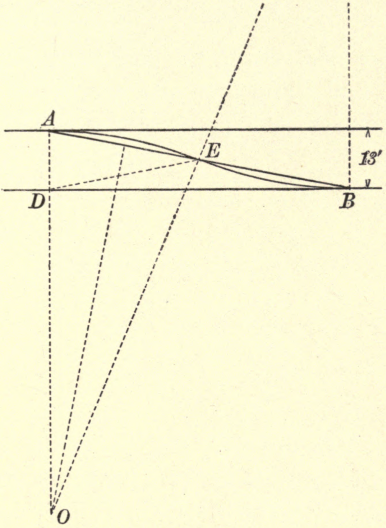


Fig. 18

nect the parallel tracks. AED and AOD are similar triangles, then

$$\frac{AE}{AD} = \frac{AO}{AE}, \quad \text{or} \quad \overline{AE}^2 = AO \times AD.$$

$$AO = \frac{\overline{AE}^2}{13} = \frac{186.4^2}{4 \times 13} = 667.$$

$$R_r = 667 \text{ feet.}$$

$$D_r = \frac{3438 \times 100}{667} = 515' = 8° 35'.$$

Problem 22. Given the conditions shown in Fig. 19, find the degree of the reversed curve, with branches of equal radii, connecting the

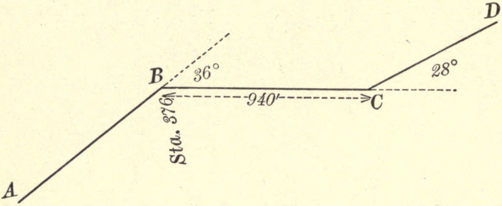


Fig. 19

tangents AB and CD, and having BC as the common tangent of the two branches, and also find the stations of the P.C., P.R.C. and P.T.

$$940 = R_r \tan 18° + R_r \tan 14°.$$

$$R_r = \frac{940}{\tan 18° + \tan 14°}$$

$$= \frac{940}{0.325 + 0.2495} = 1640 \text{ feet}$$

$$D_r = \frac{3438 \times 100}{1640} = 209' = 3° 29'.$$

Use 3° 30′.

R for 3° 30′ curve = 1637 feet.

$T_1 = 1637 \tan 18° = 532$ feet.

$$\begin{array}{lr}
 & 376 + 00 \\
 & 5 + 32 \\ \hline
\text{P.C. at} & 370 + 68 \\
 & 10 + 28.6 \\ \hline
\text{P.R.C. at} & 380 + 96.6 \\
 & 8 + 00.0 \\ \hline
\text{P.T. at} & 388 + 96.6
\end{array}$$

$$L_1 = \frac{36}{3.5} = 10 + 28.6.$$

$$L_2 = \frac{28}{3.5} = 8 + 00.$$

CHAPTER IV

THE VERTICAL CURVE

18. The vertical curves, used at sags and summits to connect adjacent grades, or used elsewhere to connect grades of the same kind, are usually parabolas. The following problem shows how the elevations on such a curve may be found.

Problem 23. Find the elevations of the stations on a ten-station vertical curve to connect the grades shown in Fig. 20.

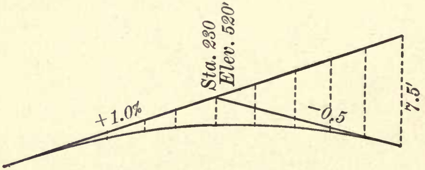


Fig. 20

The offsets from the tangent to the parabola vary as the squares of the distances from the beginning of the curve to the points where the offsets are made. The beginning of the vertical curve is at station 225 and the end is at

station 235. The elevation of station 225 is 515 and of station 235 is 517.5. The vertical offset from a point on the prolongation of the 1-per cent grade to station 235 is 7.5 feet. By the above rule the offset at station 226 is $\frac{1}{100}$ of 7.5 or 0.075. The other offsets may be found from this by the above rule.

Station.	Elevation on the straight grade.	Offset.	Elevation on the curve.
225	515.00	....	515.000
226	516.00	0.075	515.925
227	517.00	0.300	516.700
228	518.00	0.675	517.325
229	519.00	1.200	517.800
230	520.00	1.875	518.125
231	521.00	2.700	518.300
232	522.00	3.675	518.325
233	523.00	4.800	518.200
234	524.00	6.075	517.925
235	525.00	7.500	517.500

The setting for the offsets is as follows: Set the right index of the slide opposite 75 on the right-hand *A* scale, and opposite 2, 3, 4, etc., on the *C* scale read 3, 675, 12, etc., on the *A* scale. By rough figuring obtain the positions of the decimal points.

A check on the elevation of station 230 on the curve may be found from the principle

that the parabola is midway between its chord and its vertex at its middle point. The elevation of the chord at its middle point is the mean of the end elevations The elevation of the middle of the curve is the mean of the elevation of the middle of the chord and the elevation of the vertex. Applying this to the above problem, the elevation of the middle of the chord is $\frac{515 + 517.5}{2} = 516.25$ and the elevation of the middle of the curve is $\frac{516.25+520}{2}$ $= 518.125$, which checks the elevation found.

By the extension of this principle, the elevations of points on the vertical curve may be found or a parabola may be laid out.

CHAPTER V

Turnouts

19. Fig. 21 is a line diagram representing a split switch turnout, the one now most commonly used. *AH* and *LK* are the movable

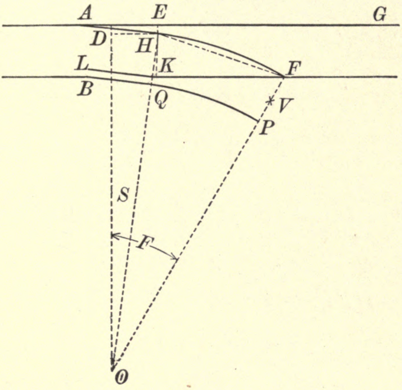


Fig. 21

rails. The rails *AG* and *BP* are continuous. *F* is the point of the frog. The rails *HF* and *QP* are curved and the rail *AH*, when the turnout is set for the branch or siding, is tan-

gent to the rail HF. AH is the switch rail, l, and HE is equal to the amount of its throw, t.

EAH, the switch angle, $= S$.

$$\text{Sin } S = \frac{EH}{AH} = \frac{t}{l}.$$

AB, the gage of the track, $= g$.

The standard gage is 4′ 8½″.

$FOD = F$, the frog angle.

$OV = R$, the radius of the turnout curve.

$(R + \frac{1}{2}g)\cos S - (R + \frac{1}{2}g)\cos F = g - t.$

$$R + \tfrac{1}{2}g = \frac{g - t}{\cos S - \cos F},$$

or

$$R = \frac{g - t}{\cos S - \cos F} - \tfrac{1}{2}g.$$

The distance BF is called the lead, L.

$$L = BK + KF.$$

Angle $\quad HFK = F - \frac{1}{2}\overline{FOE} = F - \frac{1}{2}(F - S)$

$$= \tfrac{1}{2}(F + S).$$

$$L = l + \frac{HK}{\tan\frac{1}{2}(F+S)} = l + \frac{g-t}{\tan\frac{1}{2}(F+S)}. \qquad (12)$$

$$HK = HF \tan\tfrac{1}{2}(F + S).$$

$$HF = 2(R + \tfrac{1}{2}g)\sin\tfrac{1}{2}(F - S).$$

$$\frac{g - t}{\tan\frac{1}{2}(F+S)} = 2(R+\tfrac{1}{2}g)\sin\tfrac{1}{2}(F-S). \qquad (13)$$

Fig. 22 shows the form of the frog. The number of the frog is $\frac{FR}{MN}$, altho $\frac{FN}{MN}$ is sometimes used.

$$\operatorname{Cot} \tfrac{1}{2} F = \frac{FR}{\frac{MN}{2}} = 2\frac{FR}{MN} = 2n. \quad . \quad (14)$$

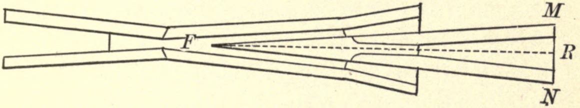


Fig. 22

Where n is the number of the frog,

$$\tan \tfrac{1}{2} F = \frac{1}{2n}. \quad . \quad . \quad . \quad (15)$$

Problem 24. Find the radius and the lead of a split switch turnout using a No. 9 frog, 18-foot switch rail, a $5\frac{1}{2}$-inch throw and standard gage track.

$$\operatorname{Sin} S = \frac{t}{l} = \frac{5.5}{12 \times 18} = \frac{5.5}{216} = 0.0254.$$

$$S = 1^\circ\ 28'.$$

$$\operatorname{Tan} \tfrac{1}{2} F = \frac{1}{2n} = \frac{1}{2 \times 9} = \frac{1}{18} = 0.0556.$$

For small angles sines and tangents are practically equal, and for all angles less than

$5° 45'$ the sine in place of the tangent scale of the slide rule should be used for tangents of such angles.

By use of the S scale of the slide, $\frac{1}{2} F = 3° 11'$ or $F = 6° 22'$.

Setting for these is: Opposite right and left indices of the A scale set the indices of the slide reversed and opposite 0.0254 on the A scale read $1° 27'$ on the S scale, also opposite 0.0556 on the A scale read $3° 11'$ on the S scale.

$$L = l + \frac{g - t}{\tan \frac{1}{2}(F + S)} = 18 + \frac{51}{12 \tan 3° 55'}$$

$$= 18 + \frac{51}{0.82}.$$

$$L = 18 + 62.2 = 80.2 \text{ feet.}$$

$$\frac{g - t}{\sin \frac{1}{2}(F + S)} = 2\left(R + \tfrac{1}{2} g\right) \sin \tfrac{1}{2}(F - S).$$

$$R + \tfrac{1}{2} g = \frac{51}{24 \sin 3° 55' \sin 2° 27'} = 728 \text{ feet.}$$

$$R = 725.65 \text{ feet.}$$

20. Where the turnout leaves the main track on a curve, the radius and the lead may be found by the precise or approximate method following.

Precise method:

1st. When the turnout is on the inside of the main track curve. In Fig. 23, O is the center of the main curve, C is the center of the turnout curve, F is the frog point and AB is the gage of the track.

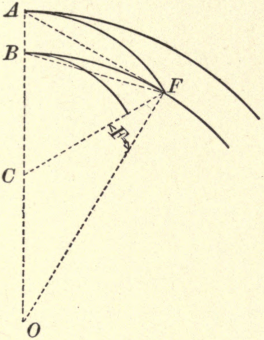


Fig. 23

In the triangle AFO,

$$OA = R_m + \frac{g}{2}, \ \ OF = R_m - \frac{g}{2},$$

$$\frac{\tan\frac{1}{2}(OFA - FAO)}{\tan\frac{1}{2}(OFA + FAO)} = \frac{R_m + \frac{g}{2} - \left(R_m - \frac{g}{2}\right)}{R_m + \frac{g}{2} + \ R_m - \frac{g}{2}} = \frac{g}{2R_m},$$

$$\frac{\tan \frac{1}{2} F}{\cot \frac{1}{2} O} = \frac{g}{2 R_m},$$

or
$$\tan \tfrac{1}{2} O = \frac{g \cot \frac{1}{2} F}{2 R_m},$$

$$\tan \tfrac{1}{2} O = \frac{gn}{R_m}. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (16)$$

In the triangle BCF, $BC = R_t - \dfrac{g}{2}$

and
$$FC = R_t + \frac{g}{2},$$

$$\frac{\tan \frac{1}{2} (CBF - BFC)}{\tan \frac{1}{2} (CBF + BFC)} = \frac{R_t + \frac{g}{2} - \left(R_t - \frac{g}{2}\right)}{R_t + \frac{g}{2} + R_t - \frac{g}{2}} = \frac{g}{2R_t}.$$

$$\frac{\tan \frac{1}{2} F}{\cot \frac{1}{2} (F + O)} = \frac{g}{2 R_t},$$

or $R_t = \dfrac{g \cot \frac{1}{2} F}{2 \tan \frac{1}{2} (F + O)} = \dfrac{gn}{\tan \frac{1}{2} (F + O)}.$ (17)

2nd. When the turnout is on the outside of the main track curve. In Fig. 24, solving the triangle AFO by the method used above,

$$\tan \tfrac{1}{2} O = \frac{gn}{R_m}.$$

Then solving the triangle BCF,

$$R_t = \frac{gn}{\tan \frac{1}{2} (F - O)}.$$

In either case, $L_t = 2\,R_m \sin \frac{1}{2}\,O,$

or $L_t = 2\,R_t \sin \frac{1}{2}\,(F \pm O).$

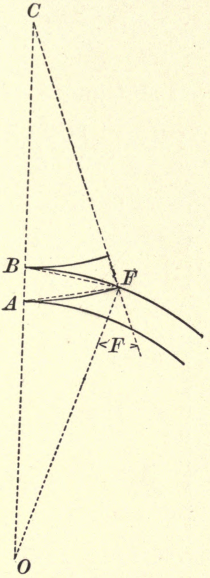


Fig. 24

21. Fig. 25 shows a turnout from a straight track where a stub switch is used. BF is the lead and AB is the gage.

$$L_s = BF = AB \cot \tfrac{1}{2}\,F = 2\,gn. \quad . \quad (18)$$

Make $BC = g$. Then the triangles AFC and AOF are similar and $AO : AF :: AF : AC$

$$AC \times AO = \overline{AF}^2.$$

$$2\,g\left(R_s + \frac{g}{2}\right) = \overline{AF}^2, \quad \text{or} \quad 2\,gR_s = \overline{AF}^2 - g^2,$$

$$2\,gR_s = \overline{BF}^2 = 4\,g^2n^2, \text{ or } \qquad R_s = 2\,gn^2. \quad (19)$$

$$R_s = gn \cot \tfrac{1}{2} F = \frac{gn}{\tan \frac{1}{2} F}. \quad . \quad . \quad . \quad (20)$$

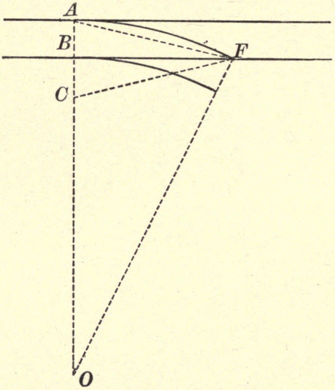


Fig. 25

22. The approximate method for turnout from curved main track.

Let R_s = radius of turnout from a straight track using a given number of frog.

R_t = radius of turnout from curved track using same number of frog.

R_m = radius of curved main track.

From Equations (16), (17) and (20), for the condition where the turnout is on the inside of the curved main track,

$$R_s : R_m : R_t :: \frac{gn}{\tan \frac{1}{2} F} : \frac{gn}{\tan \frac{1}{2} O} : \frac{gn}{\tan \frac{1}{2}(F + O)}.$$

As F and O are small angles, the tangents may be taken equal to the arcs of unit radius, or

$$R_s : R_m : R_t :: \frac{1}{F} : \frac{1}{O} : \frac{1}{F + O}.$$

The degrees of curves are practically inversely as their radii, or

$$D_s : D_m : D_t :: F : O : F + O.$$

By composition $(D_s + D_m) : D_t :: F + O : F + O$.

Hence $\qquad D_t = D_s + D_m$.

Where the turnout is on the outside of the curved main track, a similar proportion applies.

$$D_s : D_m : D_t :: F : O : F - O.$$

By division

$$D_s - D_m : D_t :: F - O : F - O.$$

Hence $\qquad D_t = D_s - D_m$.

The condition of $D_t = D_m - D_s$ may arise as shown in Fig. 26, where O is the center of the main track curve and C is the center of the turnout curve.

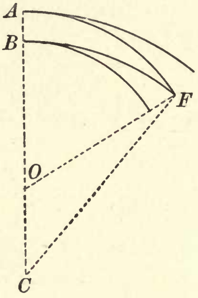


Fig. 26

In either case, $L_t = 2\,R_t \sin \frac{1}{2}\,(F \pm O)$, and as F and O are small angles, the tangents and sines are about equal or

$$L_t = 2\,R_t \tan \tfrac{1}{2}\,(F \pm O),$$

$$R_t = \frac{gn}{\tan \frac{1}{2}\,(F \pm O)}.$$

Hence $$L_t = 2\,gn.$$

Problem 25. Find the radius and the lead of a turnout on the inside of a 4° main track of standard gage, using a No. 9 frog. Both

the precise and approximate methods of solution follow.

Precise method:

$$\tan \tfrac{1}{2} O = \frac{gn}{R_m},$$

$$\tan \tfrac{1}{2} O = \frac{4.708 \times 9}{1432},$$

$$R_m = \frac{3438 \times 100}{240} = 1432.$$

$$\tan \tfrac{1}{2} O = 0.0296.$$

Solving by the slide rule using the sine scale as described in Problem 24, $\frac{1}{2}\, O = 1^\circ\, 42'$ and $O = 3^\circ\, 24'$. $F = 6^\circ\, 22'$ as in Problem 24.

$$R_t = \frac{gn}{\tan \frac{1}{2}(F + O)} = \frac{4.709 \times 9}{\tan 4^\circ\, 53'} = 496 \text{ feet.}$$

(Method of solution as given in Problem 24.)

$$D_t = \frac{3438 \times 100}{496} = 692' = 11^\circ\, 32'.$$

$$L_t = 2\, R_t \sin \tfrac{1}{2}(F + O) = 2 \times 496 \times \sin 4^\circ\, 53'$$
$$= 84.6 \text{ feet.}$$

Approximate method:

$$D_t = D_m + D_s.$$

$$D_s = \frac{5730}{2\, gn^2} = \frac{2865}{4.708 \times 9^2} = 7.48^\circ = 7^\circ\, 29'$$

$$D_m \qquad = \ \ 4^\circ\ \ 0'$$
$$D_t \qquad = \overline{11^\circ\ 29'}$$
$$L_t = 2\,gn = 2 \times 4.708 \times 9 = 84.6 \text{ feet.}$$

The use of the slide rule for the solution has been given for similar formulas.

23. Fig. 27 shows in outline the connection between a parallel siding and the main track by a turnout and a connecting curve to which the following problem applies.

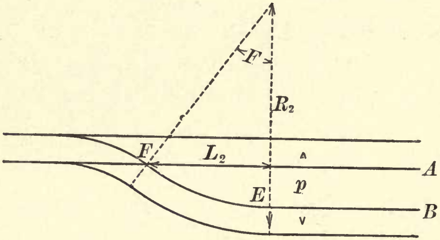


Fig. 27

Problem 26. Find R_2, D_2 and L_2, where the distance between track centers is 13 feet and a No. 9 frog is used.

FA and *EB* may be considered as the rails of a track. Then the curve *EF* may be considered as the curve part of a turnout. The gage for this is $p - g$.

Then $$R_2 - \frac{p}{2} = (p - g)\,2\,n^2.$$

$$R_2 - 6.5 = (13 - 4.71)\,2 \times 9^2$$
$$= 8.29 \times 2 \times 9^2 = 1340.$$
$$R_2 = 1346.5 \text{ feet.}$$

$$D_2 = \frac{3438 \times 100}{1346.5} = 255' = 4^\circ\,15'.$$

F for a No. 9 frog may be found from

$$\tan \tfrac{1}{2} F = \frac{1}{2\,n} = \frac{1}{18} = 0.055.$$

Using tangent = sine,

$$\frac{F}{2} = 3^\circ\,11', \quad \text{or} \quad F = 6^\circ\,22'.$$

$$L_2 = \left(R_2 - \frac{g}{2}\right) \sin F = 1344.3 \sin 6^\circ\,22'$$
$$= 149 \text{ feet.}$$

24. Fig. 28 shows a connection between two parallel tracks by the use of two similar turn-

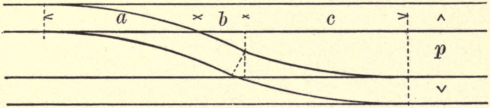


Fig. 28

outs and a piece of straight track whose center line is a common tangent to the center lines

of the curves of the turnouts. The following problem applies to this form of a connection.

Problem 27. Find the distances a, b and c for the connection between two parallel straight tracks of standard gage, with 13 feet between track centers and with No. 9 frogs used in the turnouts.

$F = 6° \, 22'$ (from Problem 26).

$a = c = 2\,gn = 9.42 \times 9 = 84.7$ feet.

$$b = \frac{p - g - g \sec F}{\tan F} = \frac{8.292 - 4.71 \sec 6° \, 22'}{\tan F}$$

$$= \frac{8.292 - \dfrac{4.71}{\cos 6° \, 22'}}{\tan F} = \frac{8.292 - 4.73}{\tan 6° \, 22'}$$

$= 31.9$ feet.

25. Fig. 29 shows the connection between the parallel siding, on the outside of the curved main track, and the main track by a turnout and a connecting curve. Solving the triangle *OBG*,

$$\frac{\tan \frac{1}{2} (OBG - BGO)}{\tan \frac{1}{2} (OBG + BGO)} = \frac{R_m + p - \frac{g}{2} - \left(R_m + \frac{g}{2}\right)}{R_m + p - \frac{g}{2} + R_m + \frac{g}{2}}.$$

$$\frac{\tan \frac{1}{2} F}{\cot \frac{1}{2} O} = \frac{p - g}{2\,R_m + p} \text{ or } \tan \frac{1}{2} O = \frac{(p - g)\,n}{R_m + \frac{p}{2}}.$$

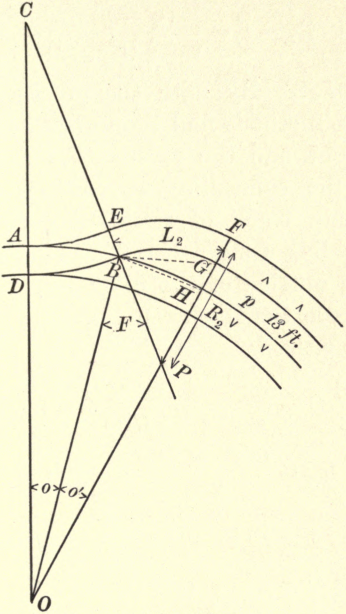


Fig. 29

Solving the triangle PBH,

$$\frac{\tan\frac{1}{2}(BHP - PBH)}{\tan\frac{1}{2}(BHP + PBH)} = \frac{R_t - \frac{g}{2} - \left(R_t - p + \frac{g}{2}\right)}{R_t - \frac{g}{2} + \ R_t - p + \frac{g}{2}}$$

$$\frac{\tan\frac{1}{2}F}{\cot\frac{1}{2}(F+O)} = \frac{p-g}{2R_t - p},$$

or
$$R_t - \frac{p}{2} = \frac{(p-g)\,n}{\tan \frac{1}{2}(F+O)}.$$

Problem 28. By both the precise and approximate methods find the radius and lead of the turnout and the radius (R_2) and length (L_2) of the connecting curve to connect a 4° 30′ main track of standard gage with a parallel siding using a No. 9 frog. The siding is on the outside of the main track, with 13 feet between the track centers.

Precise method:

$$\tan \frac{1}{2} O = \frac{gn}{R_m} = \frac{4.708 \times 9}{1270} = 0.0333,$$

$$R_m = \frac{3438 \times 100}{270} = 1270 \text{ feet}.$$

Solving by the slide rule using the sine scale,

$$\tfrac{1}{2} O = 1^\circ\, 54' \text{ and } O = 3^\circ\, 48'$$
$$F = 6^\circ\, 22'$$
$$F - O = 2^\circ\, 34'$$
$$\tfrac{1}{2}(F - O) = 1^\circ\, 17'$$

$$R_t = \frac{gn}{\tan \frac{1}{2}(F-O)} = \frac{4.708 \times 9}{\tan 1^\circ\, 17'} = 1890 \text{ feet},$$

$$D_t = \frac{3438 \times 100}{1890} = 182',$$

$$D_t = \qquad 3^\circ\, 02',$$

$$L_t = \frac{100 \times 3° \, 48'}{4° \, 30'} = \frac{100 \times 228}{270} = 84.6',$$

$$\tan \tfrac{1}{2} O' = \frac{(p - g)n}{R_m + \dfrac{p}{2}} = \frac{8.29 \times 9}{1276.5} = 0.0584.$$

Solving by the slide rule using the sine scale,

$$\begin{aligned} \tfrac{1}{2} O &= 3° \, 20' \\ O &= 6° \, 40' \\ F &= 6° \, 22' \\ \hline F + O &= 13° \;\; 2' \\ \tfrac{1}{2}(F + O) &= 6° \, 31' \end{aligned}$$

$$R_2 - \frac{p}{2} = \frac{(p - g)\, n}{\tan \frac{1}{2}(F + O)} = \frac{8.29 \times 9}{\tan 6° \, 31'} = 653$$

and

$$R_2 = 659.5 \text{ feet.}$$

$$D_2 = \frac{3438 \times 100}{659.5} = 522 = 8° \, 42'.$$

$$L_2 = \frac{100 \times 13° \, 02'}{8° \, 42'} = \frac{100 \times 782}{522} = 150 \text{ feet.}$$

Approximate method:

$$\begin{aligned} \text{By Problem 25, } D_s &= 7° \, 29' \\ D_m &= 4° \, 30' \\ \hline D_t &= 2° \, 59' \end{aligned}$$

$$L_t = 2\, gn = 84.6 \text{ feet.}$$

Let R_3 = radius of the connecting curve if the tracks were straight.

$$R_3 - \frac{p}{2} = 2\,(p-g)\,n^2 = 2 \times 8.29 \times 9^2 = 1340 \text{ feet.}$$

$$R_3 = 1346.5 \text{ feet}$$

and

$$D_3 = \frac{3438 \times 100}{1346.5} = 255',$$

$$D_3 = 4^\circ\ 15'$$

$$D_m = 4^\circ\ 30'$$

$$\overline{D_2 = 8^\circ\ 45'}$$

$$R_2 = \frac{3438 \times 100}{525} = 655 \text{ feet.}$$

$$L_2 = 2\,(p-g)\,n = 149 \text{ feet.}$$

Problem 29. By the precise and approximate methods find the radius and the length of the connecting curve for a parallel siding on the inside of a 4° main track of standard gage, with 13 feet between the track centers and using a No. 9 frog.

Precise method:

$$\tan \tfrac{1}{2}\,O = \frac{(p-g)\,n}{R_m - \dfrac{p}{2}} = \frac{8.29 \times 9}{1425.5} = 0.0523.$$

Using the sine for the tangent

$$\tfrac{1}{2}\,O = 3°\ 00', \text{ or } O = 6°\ 00'.$$

F as found in previous problem $= 6°\ 22'$,

$$F - O = 22'.$$

$$R_2 - \frac{p}{2} = \frac{(p-g)\,n}{\tan \frac{1}{2}\,(F-O)} = \frac{8.29 \times 9}{\tan 0°\ 11'},$$

$$\tan 11' = \text{approximately } 11 \times 0.00029$$
$$= 0.00319.$$

$$R_2 - \frac{p}{2} = \frac{8.29 \times 9}{0.00319} = 23{,}390,$$

or

$$R_2 = 23{,}384 \text{ feet.}$$

$$D_2 = \frac{3438 \times 100}{23{,}384} = 15' \text{ (approximately).}$$

$$L_2 = \frac{22'}{15'} = 147 \text{ feet.}$$

Approximate method:

$$R_s - \frac{p}{2} = 2\,(p-g)n^2 = 2 \times 8.29 \times 9^2 = 1340 \text{ feet.}$$

$$R_s = 1346.5 \text{ feet.}$$

$$D_s = \frac{3438 \times 100}{1346.5} = 255' = 4°\ 15'.$$

$$D_2 = 4°\ 15' - 4° = 15'.$$

$$L_2 = 2\,(p-g)\,n = 149 \text{ feet.}$$

26. "Y" curves are used to connect the main tracks and branch tracks, so that the trains coming in either direction may go from the main tracks to the branch tracks without backing. A "Y" and a branch track may also be used for reversing the direction of the train on the main track.

Problem 30. A branch track leaves a straight main track at station 322 + 40. The radius of the branch curve is 762.7 feet. Find the P.C. on the main track and the P.T. on the branch track of a "Y" curve, of 573.7 feet radius, that will connect the main and branch tracks. A is at station 322 + 40. O is the center of the branch curve. C is the center, L is the P.C., and M is the P.T. of the "Y" curve.

Let α = central angle of the branch track from its P.C. to the P.T. of the "Y," B = central angle of the "Y" curve, R_b = the radius of the branch curve and R_y = radius of the "Y" curve.

$$\cos \alpha = \frac{R_b - R_y}{R_b + R_y} = \frac{762.7 - 573.7}{762.7 + 573.7}$$

$$= \frac{189}{1336.4} = 0.142.$$

By the use of the sine scale of the slide rule,

$$\text{sine of } (90° - \alpha) = 0.142;\ 90° - \alpha = 8°\ 10'$$

$$\alpha = 81°\ 50'.$$

$$B = 180° - \alpha = 180° - 81°\ 50' = 98°\ 10'.$$

l = distance from P.C. of the branch to the P.C. of the "Y" measured along the main track.

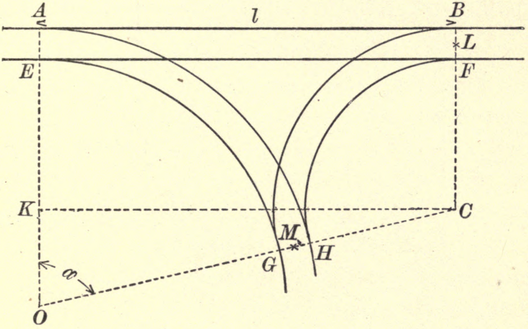


Fig. 30

$$l = 1336.4 - \frac{189^2}{2 \times 1336.4} = 1322.9 \text{ feet.}$$

$$\begin{array}{r} 322 + 40 \\ 13 + 22.9 \\ \hline 335 + 62.9 \end{array}$$

P.C.$_y$ is at 335 + 62.9

$$D_b = \frac{3438 \times 100}{762.7} = 450' = 7° 30',$$

$$L_b = \frac{81° 50' \times 100}{7° 30'} = 1090 \text{ feet},$$

or P.T.$_y$ is at station $10 + 90$ on the branch track.

Problem 31. Using the same curves for the branch and "Y" tracks as in No. 30, but having

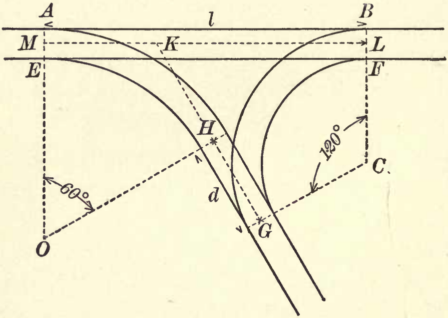


Fig. 31

a 60° central angle for the branch curve and the P.T. of "Y" coming beyond the P.T. of the branch and on the tangent to the branch curve at its P.T., find the P.C. on the main track and the P.T. on the branch track for the "Y" curve.

Using letters which represent the same things as in Problem 30,

$$l = 762.7 \tan \frac{60°}{2} + 573.7 \tan \frac{120°}{2},$$

$$l = 762.7 \tan 30° + 573.7 \tan 60°,$$

$$l = 762.7 \tan 30° + \frac{573.7}{\tan 30°},$$

$$l = 441 + 994 = 1435 \text{ feet.}$$

Settings. Reverse the slide, set right index of T scale of the slide opposite 762.7 on the D scale and opposite 30° on the T scale read 441 on the D scale.

Set 30° on the T scale opposite 573.7 on the D scale, and opposite the right index of the T scale read 994 on the D scale.

Let d = the distance from the P.T. of the branch curve to the P.T. of the "Y" curve measured along the tangent to the branch curve thru its P.T.

$$d = R_y \tan \frac{120°}{2} - R_b \tan \frac{60°}{2}$$

$$= \frac{573.7}{\tan 30°} - 762.7 \tan 30° = 994 - 441$$

$$= 553 \text{ feet.}$$

$$\begin{array}{lr} & 322 + 40 \\ l = & 14 + 35 \\ \hline \text{P.C.}_y \text{ at} & 336 + 75 \end{array}$$ on the main track.

$$D_b = \frac{3438 \times 100}{762.7} = 450' = 7° \, 30',$$

$$L_b = \frac{60°}{7° \, 30'} \times 100 = 800 \text{ feet}, \qquad 8 + 00$$

$$\begin{array}{rr} d = & 5 + 53 \\ \hline \text{P.T.}_y \text{ at} & 13 + 53 \end{array}$$

on the branch track.

CHAPTER VI

THE EASEMENT CURVE

27. The purpose of the easement curve, connecting the tangent and the circular arc, is to produce a gradually increasing centrifugal force that may be balanced by a gradually increasing centripetal force produced by the gradual elevation of the outer rail of the easement curve part of the track. The ordinary form of the easement curve is the spiral or a similar curve.

From the above it is seen that the length of the spiral is the distance in which the total amount of the elevation of the outer rail is gained. In the best practice the rate, by which the elevation of the outer rail is gained, is a function of the speed of the train. It has been found that a gain of $1\frac{1}{6}$ inches per second is not felt by a passenger in the train. Even a gain of two inches per second does not produce a disagreeable effect.

Let e = the elevation of the outer rail, in inches, necessary for the circular curve.

Let v = the velocity of the train in feet per second.

S = the velocity of train in miles per hour.

l_c = the length of the spiral.

D_c and R_c = the degree and the radius of the circular curve.

r = the rate at which e is gained in inches per second.

C = the centrifugal force of a car on the circular curve.

W = the weight of the car.

G = the gage of the track.

g = the acceleration due to gravity.

Then from mechanics,

$$C = \frac{Wv^2}{gR_c}.$$

From Fig. 32,

$$\frac{C}{W} = \frac{BD}{AD} = \frac{BD}{AB} \text{ (approximately).}$$

$$\frac{\frac{Wv^2}{gR_c}}{W} = \frac{e}{G} \quad \text{or} \quad e = \frac{Gv^2}{gR_c}, \quad . \quad . \quad . \quad (21)$$

$$v = 1.47\,S, \qquad G = 4.71,$$

$$g = 32.2, \text{ and } R_c = \frac{5730}{D_c}.$$

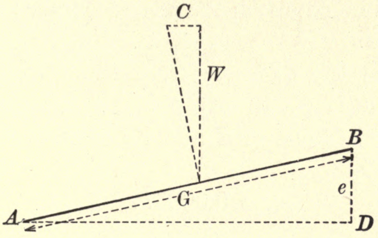


Fig. 32

Substituting these values in Equation (21), reducing and expressing the result in inches, $e = 0.00067\,S^2D_c$. At a rate of gain, for e, of r inches per second,

$$l_c = \frac{e}{r} \times v = \frac{0.00067\,S^2D_c}{r}\,v.$$

Substituting for v its value in terms of S,

$$l_c = \frac{0.00067\,S^2D_c}{r} \times 1.47\,S = \frac{S^3D_c}{1000\,r}\text{ (approx.).} \tag{22}$$

For a rate of $1\frac{1}{6}$ inches per second,

$$l_c = \frac{S^3D_c}{1200}\text{(approx.)}$$

For a rate of 2 inches per second,

$$l_c = \frac{S^3D_c}{2000}.$$

28. The equation for a true spiral is derived as follows:

If l is the distance from the beginning of the spiral, P.S., to any point, P, on the spiral where its radius is R and its degree of curvature is D,

$$e_P = 0.00067\, S^2 D$$

and

$$l = \frac{0.00067\, S^2 \times 5730\, v}{r \times R} = \frac{5.73\, S^3}{r \times R} \text{ (approx.)}.$$

$$Rl = \frac{5.73\, S^3}{r}.$$

In a similar way it is readily shown that

$$R_c l_c = \frac{5.73\, S^3}{r}.$$

Hence

$$Rl = R_c l_c \text{ or } \frac{D}{l} = \frac{D_c}{l_c}. \quad . \qquad (23)$$

This is the equation for a true spiral and should be used for very long curves which partake more of the nature of a curve compounded many times than of an easement curve which connects a tangent and a circular arc. This equation shows that the radius of the spiral varies inversely as the distance from the beginning of the spiral, or its degree of curva-

ture varies uniformly with the distance from the beginning of the spiral.

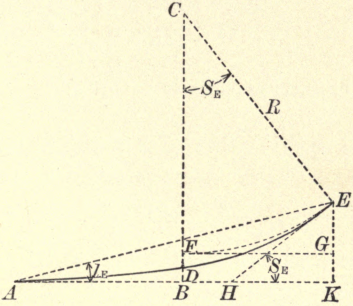


Fig. 33

In Fig. 33, ADE represents part of an easement curve or spiral, beginning at A, the point of spiral. Let l = length of spiral from A to E, S_E = the spiral angle for point E and i_E = its deflection angle from tangent AK.

29. To find the value of S_E.

From the circular curve EF of radius R and degree D,

$$s_E = \frac{l}{2 \times 100} D^\circ \left(EF = \frac{l}{2}, \text{ approx.}\right).$$

$$s_E = \frac{l}{2 \times 100} \frac{5730}{R}.$$

$$s_E = \frac{l}{2\,R} 57.3^\circ \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (24)$$

Equation (24) gives the value of S_E in degrees. In length of arc of unit radius

$$s_E = \frac{l}{2\,R} \quad . \quad . \quad . \quad . \quad . \qquad (25)$$

From Equation (23) $R = \dfrac{R_c l_c}{l}$;

substituting this value of R in Equation (25)

$$s_E = \frac{l^2}{2\,R_c l_c} \quad . \quad . \quad . \quad . \quad . \qquad (26)$$

30. To find the relation between deflection angles from the tangent at the point of spiral to points on spiral.

Assuming that the part of the spiral from A to E is a circular arc of $D°$ curvature (practically true if AE is a very small part of the spiral), the deflection

$$i_E = \frac{l}{100}\,\frac{D°}{2} \quad \text{or} \quad i_E = \frac{l}{200}\,D° \quad . \quad . \qquad (27)$$

The curvature of the spiral at point $D = \dfrac{D°}{2}$,

where $D°$ is the curvature of the spiral at E and $AD = \dfrac{AE}{2} = \dfrac{l}{2}$.

Considering the spiral as a circular curve of $\dfrac{D°}{2}$ from A to D,

$$i_D = \frac{\frac{l}{2}}{100}\frac{\frac{D^\circ}{2}}{2} = \frac{l}{800}D^\circ, \quad . \quad . \quad . \quad (28)$$

hence
$$\frac{i_E}{i_D} = \frac{\frac{l}{200}D^\circ}{\frac{l}{800}D^\circ} = \frac{4}{1}. \quad . \quad . \quad . \quad . \quad . \quad (29)$$

Hence deflections from the tangent at the P.S. to points on the spiral vary as the squares of the distances from the P.S. to the points on the spiral.

31. To find the relation between the offsets from the tangent thru the P.S. to points on the spiral.

$$p_1 = DB = \frac{l}{2}\sin i_D,$$

$$p_2 = EK = l \sin i_E,$$

$$\frac{p_2}{p_1} = \frac{l \sin i_E}{\frac{l}{2}\sin i_D}.$$

Since the sines and arcs for small angles are proportional, $\frac{p_2}{p_1} = \frac{2\, i_E}{i_D}$ and substituting the value of $\frac{i_E}{i_D}$ from Equation (29),

$$\frac{p_2}{p_1} = \frac{8}{1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

Hence offsets from the tangent thru the P.S. to points on the spiral vary as the cubes of the distances to the said points from the point of spiral.

32. To find the value of the deflections from the tangent thru the P.S. to points on the spiral.

Since the spiral changes in curvature uniformly with the distance, the amount that it will deflect from the tangent for the distance $AD = \frac{l}{2}$ is the same that it deflects from the circular arc for the distance $EF = \frac{l}{2}$ or $FD = DB$.

Hence $FB = 2\,DB = 2\,p_1 = \frac{p_2}{4}$.

From the circular curve FE,

$$EG = \frac{\left[\frac{l}{2}\right]^2}{2R} = \frac{l^2}{8\,R} = p_2 - 2\,p_1 = \frac{3\,p_2}{4},$$

or

$$\frac{l^2}{8\,R} = \frac{3\,p_2}{4},$$

then

$$p_2 = \frac{4\,l^2}{24\,R} = \frac{l^2}{6\,R}. \quad . \quad . \quad . \quad (31)$$

$$p_2 = l \sin i_E.$$

For small angles sines and arcs of unit radii are equal, then

$$p_2 = li_E = \frac{l^2}{6\,R},$$

or $$i_E = \frac{l}{6\,R} \quad \ldots\ldots \quad (32)$$

$$R = \frac{R_c l_c}{l},$$

hence $$i_E = \frac{l^2}{6\,R_c l_c}. \quad \ldots\ldots \quad (33)$$

$$s_E = \frac{l^2}{2\,R_c l_c},$$

hence $$i_E = \frac{s_E}{3}.$$

Since E may be taken as any point on the spiral,

$$i = \frac{s}{3}. \quad \ldots\ldots \quad (34)$$

33. To show $p = \dfrac{l^2}{24\,R}$,

$$p = FB = \frac{p_2}{4}, \text{ but } p_2 = \frac{l^2}{6\,R} \text{ from Eq. (31),}$$

hence $$p = \frac{l^2}{24\,R}. \quad \ldots\ldots \quad (35)$$

34. To find a value for the deflection angle, i, to any point on the spiral.

Let l = distance from P.S. to the point where the deflection is i.

N = number of chords from P.S. to the same point.

C = rate of change in curvature of the spiral per chord length.

$i = lNC \times 0.1'$* giving i in minutes.

From Equation (32)

$$i = \frac{l}{6R},\ R = \frac{5730}{D}\ \text{(approximately)}.$$

* From Kellogg's Transition Curve by N. B. Kellogg, C. E.

NOTE. If a spiral of *six* chords in length is always used the deflection for the end of the first chord is $\frac{l_c}{6} \times 1 \times \frac{D_c}{6} \times 0.1$ in minutes $= \frac{1}{60}\frac{l_c}{6} D_c$ or in seconds is $l_N\ D_c$, where l_N is the length of a chord, which, expressed as a rule, is:

RULE. For a six-chord spiral the deflection for the end of the first chord in seconds is the length of the chord in feet multiplied by the degree of the circular curve.

Deflections for the ends of the other chords may be found by the rule that the deflections vary as the square of the distances from the point of spiral; i.e., the deflection for the end of the second chord is four times that for the end of the *first chord* and for the end of the third, nine times that for the end of the first, etc.

Hence

$$i = \frac{l\,D}{6 \times 5730}. \quad D = NC,$$

hence

$$i = \frac{lNC}{6 \times 5730}$$

in length of arc of unit radius, and multiplying by 57.3 gives i in degrees,

$$i = \frac{lNC}{6 \times 100}.$$

Multiplying this value of i by 60 gives i in minutes.

$$i = lNC \times 0.1. \quad . \quad . \quad . \quad . \quad (36)$$

In Equation (36) l is expressed in feet, N in chord lengths and C in degrees. N may have a fractional value.

35. To find the distance AB in Fig. 34,

$$AB = q = \frac{l_c}{2} \text{ (approximately)}.$$

The following gives a value for q more nearly correct.

In any right-angled triangle the approximate difference between the hypotenuse and base is equal to the square of the altitude divided

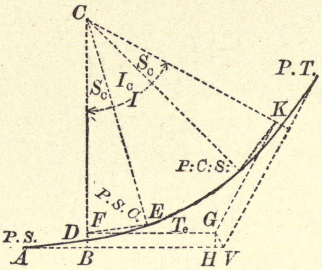


Fig. 34

by twice the known side, either base or hypotenuse, hence

$$AD - AB = \frac{\overline{BD}^2}{2\,AD}, \quad AD = \frac{l_c}{2}, \; BD = \frac{P}{2}.$$

Then $$AB = \frac{l_c}{2} - \frac{\left[\frac{P}{2}\right]^2}{\frac{2\,l_c}{2}}.$$

$$AB = \frac{l_c}{2} - \frac{P^2}{4\,l_c}. \quad . \quad . \quad . \quad . \quad (37)$$

$$P = \frac{l_c^2}{24\,R_c} \quad \text{(Equation (35))}.$$

$$q = AB = \frac{l_c}{2} - \frac{l_c^3}{4 \times 24^2 R_c^2}$$

$$= \frac{l_c}{2} - \frac{l_c^3}{48^2 R_c^2}. \quad . \quad . \quad . \quad (38)$$

36. To find the distance of the P.S. from V, the intersection of the tangents.

Ordinarily the same spiral will be used at each end of the circular curve. The distance from P.S. to V will be found under this condition.

$$AV = AB + BH + HV. \quad \text{Let } T_s = AV.$$

$$T_s = q + R_c \tan\frac{I}{2} + P\tan\frac{I}{2}. \quad . \quad . \quad (39)$$

37. To find a tangent at any point on the spiral. In Fig. 33, the angle $AEH = EHK - EAH$.

$$AEH = s_E - i_E = 3\,i_E - i_E = 2\,i_E. \quad . \quad (40)$$

Set up the transit at E and with instrument set at 0° 00′, sight to A. Transit the telescope and turn off an angle of $2\ i_E$, in the same direction as the curve is running, and the line of sight will be in a tangent to the spiral at E.

38. To find the deflection from a tangent at any point on the spiral to any other point on the spiral.

1°. In the direction from the P.S. toward the circular curve.

Let KFL be an arc of a circle of the same radius as that of the spiral at F.

Since the spiral changes in curvature uniformly with the distance, the angular amount that the spiral deflects from the curve FL is the same that it deflects from the tangent AB in a distance equal to FL, or the angle EFL equals the deflection from the tangent thru the point of spiral for a distance of FL. The angle LFM is the deflection angle for a chord of FL for the circular curve of radius R. The angle $EFM = EFL + LFM$.

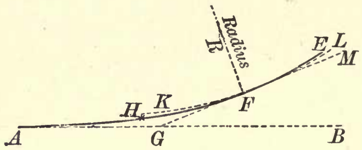


Fig. 35

Then the deflection angle from the tangent at any point on the spiral to any other point on the spiral is equal to the sum of the deflections for the spiral and for the circular arc for the length of chord between the two points on the spiral, the circular arc having the same radius as the spiral at the point thru which the tangent runs, and the spiral being run in toward the circular curve which it connects with main tangent.

2°. In the direction from the circular curve toward the tangent.

By a proof similar to that just given, it may be shown that the deflection from the tangent at any point on the spiral to any other point on the spiral is equal to the difference of the deflections for the circular arc and the spiral for the length of chord between the points on the spiral, the circular arc having the same radius as the spiral at the point thru which the tangent runs, when the spiral is run in from the circular curve toward the main tangent.

39. There are two common methods of laying out easement curves, viz.: first, by deflection angles, and second, by offsets from the tangent thru the P.S.

The following steps give the deflection angle method by the use of tables derived from the formulas and by the use of the slide rule.

1°. From the assumed speed of the train, the degree of the curve and the rate of gaining the elevation of the outer rail, find the length of the easement curve by Equation (22) or from a table or diagram made from this equation. The slide rule readily solves the equation.

2°. From tables, or by slide rule, find values of p and q. By formula or table find the tan-

gent distance, T_c, for the circular curve for a central angle equal to the deflection angle between the tangents. Find value of T_s in Equation (39).

$$T_s = q + T_c + P \tan \frac{I}{2}.$$

From station of V, intersection of tangents, subtract T_s expressed in stations and result is the station P.S.

3°. By the deflections and their corresponding distances from the P.S. run in the spiral as far as the P.S.C., where it joins the circular arc (Fig. 34).

From the deflection angle between the tangents subtract twice the spiral angle, S_c, for the spiral used, and the result is the central angle of the circular arc, from which its length may be determined.

Set the transit up at P.S.C. and run in the circular arc by deflections from the tangent thru the P.S.C., to the P.C.S., where it joins the ending spiral. Set up the transit at P.C.S. and run in the ending spiral to the P.T. by deflections from the tangent thru the P.C.S. Set up the transit at the P.T. and check work by sighting on the P.C.S. and turning off one-half of the deflection angle for the P.T.,

and if line found runs thru V the work is correct.

40. The method of laying out a circular curve with spirals at each end, the spirals to be located by offsets from tangents, is as follows:

See Fig. 34.

1°. The circular curve is to be shifted from the tangents toward the center, until the shifted P.C. is a distance of "p" from the tangent. As the curve is moved toward the center every point on the curve will move along a line parallel to the line joining the center of the curve with its vertex. The shifted P.C. will

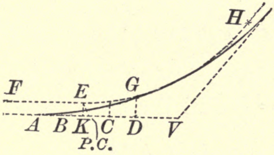


Fig. 36

be directly opposite a point on the tangent at a distance of $p \tan \frac{I}{2}$ from the original position of the P.C., measured backward along the tangent.

Set the instrument up at E, the shifted P.C., and run in the circular curve by deflections from the tangent EF parallel to AV.

2°. From the length of the spiral found as described in paragraph 39, find the value of q and p. From station of original P.C. for the circular curve subtract $q + p \tan \frac{I}{2}$ to find the station of A, the point of spiral.

The offset from the tangent AV to the middle point of the spiral is $\frac{p}{2}$. By paragraph 31, the offsets to the spiral from any point on the tangent AV is to $\frac{p}{2}$ as the cube of the distance of said point from A is to the cube of $\frac{l}{2}$.

Let $AB = \frac{l}{4}$ and the offset to the spiral at B $= b$, then $b : \frac{p}{2} :: \left(\frac{1}{4}\right)^3 : \left(\frac{1}{2}\right)^3$ or $b = \frac{p}{16}$.

Let $AC = \frac{3\,l}{4}$ and $c =$ offset at C,

then $$c = \frac{27}{16}\,p.$$

$AD = l$ and $d =$ offset at D, then $d = 4\,p$.

Find the location of A, the P.S., as described, then lay off on the tangent AB, AK, AC and AD.

At B lay off b; at K, $\frac{p}{2}$; at C, c; at D, d.

The ends of the offsets are in the spiral.

This method is not theoretically correct, as it assumes that the distance from the P.S. to any perpendicular to the tangent is the same measured along the tangent as along the spiral. However, the resulting curve is practically the same as the one found by a precise method.

41. To connect the two branches of a compound curve by a spiral, find the difference in degrees of curvature between the two branches. Then from the adopted speed, rate of elevation of the outer rail and the difference in degree of curvature, find the length of the spiral. Use the spiral given in the tables, that is practically of this length, or figure the spiral deflections, etc., by the slide rule.

The number of chords in this spiral multiplied by the change in curvature per chord must be equal to the difference in the degrees of curvature of the two parts of the compound curve.

The "p" of the spiral will be the distance GH in Fig. 37.

$AG = BH$ is equal to "q" of the spiral used.

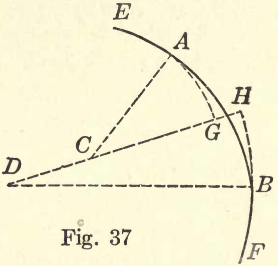


Fig. 37

Assume that the curve is to be run in from F to E. Let H be the original P.C.C. Find B on the curve FH, at a distance of "q" back from H. Set the transit at B and run in the spiral by deflections from the tangent at B, as given in paragraph 38, to A. Set the transit at A and run in the circular curve AE by deflections from the tangent at A.

42. In the tables on pages 100 and 102 are given the deflections from the tangent through the point of spiral, for different lengths of spiral and different changes in curvature per chord length for ten, twenty, thirty, forty and fifty foot chords, and also the values of p and q for spirals of different lengths and changes in

curvature per chord. From the tables, data for running in the center line spirals for street railway tracks may be found, using chords either 10 or 20 feet in length.

43. The table on page 96 shows the spirals given in the tables on pages 100 and 102 that may be used for different degrees of the circular curves.

A similar table can be made for the spirals to be used with any circular curve up to 20°.

After selecting the spiral for any particular curve the deflections for the spiral may be taken directly from the tables, or be figured by the slide rule.

44. Where a spiral of the proper length for a given circular curve cannot be taken directly from the table, the following example shows a method that may be used: It is desired to use a spiral 165 feet long for a 5° 30′ circular curve. Take from the tables the deflections for the spiral 150 feet long having a change of curvature of 1° for each 30′ chord. This leaves only the deflection for the end of the spiral to be determined. This may be found by formula (36), $i = lNC \times 0.1$. In this case $l = 165'$, $N = 5\frac{1}{2}$ and $C = 1$. $i = 1° 30.75'$.

For ease in computation, the last chord of the

Degree of circular curve.	Length of spiral.	Change of curvature		
		Per 30′ chord.	Per 40′ chord.	Per 50′ chord.
3° 00′	60′	1° 30′	....	....
“	80′	....	1° 30′	....
“	90′	1° 00′	....	....
“	100′	....	....	1° 30′
“	120′	0° 45′	1° 00′	....
“	160′	....	0° 45′	....
“	180′	0° 30′	....	....
“	200′	....	....	0° 45′
“	240′	....	0° 30′	....
“	300′	....	....	0° 30′
4° 00′	60′	2° 00′	....	....
“	80′	....	2° 00′	....
“	100′	....	....	2° 00′
“	120′	1° 00′	....	....
“	160′	....	1° 00′	....
“	180′	0° 40′	....	....
“	200′	....	....	1° 00′
“	240′	0° 30′	0° 40′	....
“	300′	....	....	0° 40′
“	320′	....	0° 30′	....
“	400′	....	....	0° 30′
5° 00′	90′	1° 40′	....	....
“	120′	1° 15′	1° 40′	....
“	150′	1° 00′	....	1° 40′
“	160′	....	1° 15′	....
“	200′	....	1° 00′	1° 15′
“	250′	....	....	1° 00′
“	300′	0° 30′	....	....
“	400′	....	0° 30′	....
“	500′	....	....	0° 30′

spiral should be taken some even fractional part of a full chord length as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ or $\frac{1}{5}$ of it. The length of the spiral expressed in chord lengths multiplied by the change in curvature per chord length must always equal the degree of the circular curve.

45. The following problem shows the method of obtaining by use of the tables the necessary quantities to locate a spiral by deflections.

Problem 32. Given two tangents intersecting at station 187 + 40, with a deflection angle of 33° 40′, to find the stations of the P.S. and the P.T., and the deflections for a 4° curve, with equal spirals at each end to connect the given tangents; the speed of train to be 40 miles per hour and the rate of superelevation to be 1.6″ per second. The length of the spiral, from the formula $l_c = \frac{S^3 D}{1000\,r}$, is 160 feet.

The curve selected from the tables may be either a 160-foot spiral consisting of four 40′ chords with a change of curvature of 1° per chord or a 180-foot spiral consisting of six 30′ chords with a change of curvature of $\frac{2°}{3}$ per chord. Assuming that the 180′ spiral is used, the following is the solution for the various quantities:

$$T_c = \frac{1733.5}{4} + 0.08 = \quad . \quad . \quad . \quad . \quad 433.5$$

$$\begin{aligned} p \tan \tfrac{1}{2} I &= 0.94 \tan 16^\circ\ 50' \\ \log\ 0.94 &= 9.973128 - 10 \\ \log 16^\circ\ 50' &= \underline{9.480801 - 10} \\ & \quad\ 9.453929 - 10 \log \text{ of } 0.28 \end{aligned}$$

$$q \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ \underline{90.0}$$

$$523.8^* \text{ feet}$$

V at station	187 + 40
	5 + 23.8
P.S. at station	182 + 16.2
	1 + 80
P.S.C. at station	183 + 96.2

Deflections from P.S. to P.S.C., for end of each chord 30 feet long, may be taken from the table. For other points deflections may be found from the formula $i = lNC \times 0.1'$.

Deflection for station 183 on the spiral is found as follows:

Distance from P.S. to 183 = 83.8′.

Distance in chords of 30′ = 2.79.

$$i = 83.8 \times 2.79 \times \tfrac{2}{3} \times 0.1 = 0^\circ\ 15.6'.$$

I_c the amount of curvature in the circular arc $= I - 2\,S_c$.

$$I_c = 33^\circ\ 40' - 7^\circ\ 12' = 26^\circ\ 28'.$$

* Results need be found only to nearest $\frac{1}{10}$ of a foot.

$$L_c = \frac{26° \, 28'}{4°} \times 100 = 661.7 \text{ feet.}$$

P.S.C. is at	183 + 96.2
	6 + 61.7
P.C.S. is at	190 + 57.9
	1 + 80.0
P.T. is at	192 + 37.9

Stations.	Deflections.	Description of curve.
182+16.2 P.S.	. . . .	$D = 4°$ R
+46.2	0–02′	$I = 33° \, 40'$
+76.2	0–08	$T_c = 433.5'$
183	0–15.6	$T = 523.8'$
+06.2	0–18	Spiral is
+36.2	0–32	180′ long
+66.2	0–50	$p = 0.94'$
+96.2 P.S.C.	1–12	$q = 90.0'$
184	0–04.6	$S_c = 3° \, 36'$
185	2–04.6	$I_c = 26° \, 28'$
186	4–04.6	$L_c = 661.7'$
187	6–04.6	
188	8–04.6	
189	10–04.6	
190	12–04.6	
190+57.9 P.C.S.	13–14	
190×87.9	0–34	
191	0–46.6	
+17.9	1–04	
+47.9	1–30	
+77.9	1–52	
192	2–05.6	
+07.9	2–10	
37.9 P.T.	2.24	

CHORD LENGTH
Change of curvature

l	1° 00′	2° 00′	3° 00′	5° 00′	8° 00′	10° 00′	1° 00′	
	Deflections.							
							p	q
10	0–01	0–02	0–03	0–05	0–08	0–10		
20	0–04	0–08	0–12	0–20	0–32	0–40	0.01	10
30	0–09	0–18	0–27	0–45	1–12	1–30	0.02	15
40	0–16	0–32	0–48	1–20	2–08	2–40	0.05	20
50	0–25	0–50	1–15	2–05	3–20	4–10	0.09	25
60	0–36	1–12	1–48	3–00	4–48	6–00	0.16	30

CHORD LENGTH
Change of curvature

l	1° 00′	2° 00′	3° 00′	5° 00′	8° 00′	10° 00′	1° 00′	
	Deflections.							
							p	q
20	0–02	0–04	0–06	0–10	0–16	0–20		
40	0–08	0–16	0–24	0–40	1–04	1–20	0.02	20
60	0–18	0–36	0–54	1–30	2–24	3–00	0.08	30
80	0–32	1–04	1–36	2–40	4–16	5–20	0.19	40
100	0–50	1–40	2–30	4–10	6–40	8–20	0.36	50
120	1–12	2–24	3–36	6–00	9–36	12–00	0.63	60
140	1–38	3–16	4–54	8–10	13–04	16–20	1.00	70

CHORD LENGTH
Change of curvature

l	30′	40′	45′	1° 00′	1° 15′	1° 30′	1° 40′	2° 00′	30′	
	Deflection angles.									
									p	q
30	0°01.5	0°–02	0°–02.25	0°–03′	0°03.75	0°04.5	0° 05′	0°06′		
60	0–06	0–08	0–09	0–12	0–15	0–18	0–20	0–24	0.03	30
90	0–13.5	0–18	0–20.25	0–27	33.75	0–40.5	0–45	0–54	0.09	45
120	0–24	0–32	0–36	0–48	1–00	1–12	1–20	1–36	0.21	60
150	0–37.5	0–50	0–56.25	1–15	1–33.75	1–52.5	2–05	2–30	0.41	75
180	0–54	1–12	1–21	1–48	2–15	2–42	3–00	3–36	0.71	90
210	1–13.5	1–38	1–50.25	2–27	3–03.75	3–40.5	4–05	4–54	1.12	105
240	1–36	2–08	2–24	3–12	4–00	4–48	5–20	6–24	1.68	120
270	2–01.5	2–42	3–02.25	4–03	5–03.75	6–04.5	6–45	8–06	2.40	135
300	2–30	3–20	3–45	5–00	6–15	7–30	8–20	10–00	3.27	150

10 FEET.
per chord.

2° 00′		3° 00′		5° 00′		8° 00′		10° 00′	
p and *q*.									
p	*q*	*p*	*q*	*p*	*q*	*p*	*q*	*p*	*q*
0.01	10	0.02	10	0.03	10	0.05	10	0.06	10
0.04	15	0.06	15	0.10	15	0.16	15	0.20	15
0.09	20	0.14	20	0.23	20	0.37	20	0.46	20
0.18	25	0.27	25	0.45	25	0.73	25	0.91	25
0.32	30	0.48	30	0.80	30	1.27	30	1.59	30

20 FEET.
per chord.

2° 00′		3° 00′		5° 00′		8° 00′		10° 00′	
p and *q*.									
p	*q*	*p*	*q*	*p*	*q*	*p*	*q*	*p*	*q*
0.05	20	0.07	20	0.12	20	0.19	20	0.23	20
0.16	30	0.24	30	0.39	30	0.63	30	0.78	30
0.37	40	0.56	40	0.93	40	1.49	40	1.86	40
0.73	50	1.09	50	1.82	50	2.91	50	3.64	50
1.25	60	1.88	60	3.13	60	5.02	59.9	6.27	59.9
2.00	70	3.00	70	5.00	70	8.00	69.9	10.00	69.8

30 FEET.
per chord.

40′		45′		1° 00′		1° 15′		1° 30′		1° 40′		2° 00′	
p and *q*.													
p	*q*	*p*	*q*	*p*	*q*	*p*	*q*	*p*	*q*	*p*	*q*	*p*	*q*
0.03	30	0.04	30	0.05	30	0.07	30	0.08	30	0.08	30	0.10	30
0.12	45	0.13	45	0.18	45	0.22	45	0.27	45	0.29	45	0.35	45
0.28	60	0.31	60	0.42	60	0.52	60	0.63	60	0.70	60	0.85	60
0.55	75	0.61	75	0.82	75	1.02	75	1.22	75	1.36	75	1.63	75
0.94	90	1.06	90	1.41	90	1.76	90	2.12	90	2.35	90	2.82	90
1.50	105	1.68	105	2.28	105	2.65	105	3.36	104.9	3.74	104.9	4.48	104.8
2.24	120	2.51	120	3.35	120	4.16	119.9	5.00	119.9	5.53	119.9	6.59	119.8
3.19	135	3.60	134.9	4.80	134.9	5.95	134.9	7.18	134.8	7.95	134.8	9.52	134.7
4.36	149.9	4.89	149.9	6.51	149.9	8.13	149.8	9.77	149.7	10.4	149.6	13.0	149.4

CHORD LENGTH

Change of curvature

l	30′	40′	45′	1° 00′	1°15′	1° 30′	1° 40′	2° 00′	30′		40′
	Deflection angles.										
									p	q	p
40	0–02	0–02.7	0–03	0–04	0–05	0–06	0–06.7	0–08			
80	0–08	0–10.7	0–12	0–16	0–20	0–24	0–26.7	0–32	0.05	40	0.06
120	0–18	0–24	0–27	0–36	0–45	0–54	1–00	1–12	0.16	60	0.21
160	0–32	0–42.7	0–48	1–04	1–20	1–36	1–46.7	2–08	0.37	80	0.50
200	0–50	1–06.7	1–15	1–40	2–05	2–30	2–46.7	3–20	0.73	100	0.97
240	1–12	1–36	1–48	2–24	3–00	3–36	4–00	4–48	1.26	120	1.68
280	1–38	2–10.7	2–27	3–16	4–05	4–54	5–26.7	6–32	2.00	140	2.64
320	2–08	2–50.7	3–12	4–16	5–20	6–24	7–06.7	8–32	2.98	160	3.97
360	2–42	3–36	4–03	5–24	6–45	8–06	9–00	10–48	4.25	180	5.65
400	3–20	4–26.7	5–00	6–40	8–20	10–00	11–06.7	13–20	5.80	199.9	7.70

CHORD LENGTH

Change of curvature

l	30′	40′	45′	1°00′	1° 15′	1° 30′	1° 40′	2°00′	30′	
	Deflection angles.									
50	0°–02′.5	0°–03.5	0°–03.75	0–05	0°–06.25	0.07.5	0–08.5	0–10	p	q
100	0–10	0–13.5	0–15	0–20	0–25	0–30	0–33.5	0–40	0.07	50
150	0–22.5	0–30	0–33.75	0–45	0–56.25	1–07.5	1–15	1–30	0.25	75
200	0–40	0–53.5	1–00	1–20	1–40	2–00	2–13.5	2–40	0.58	100
250	1–02.5	1–23.5	1–33.75	2–05	2–36.25	3–07.5	3–28.5	4–10	1.14	125
300	1–30	2–00	2–15	3–00	3–45	4–30	5–00	6–00	1.97	150
350	2–02.5	2–43.5	3–03.75	4–05	5–06.25	6–07.5	6–48.5	8–10	3.12	175
400	2–40	3–33.5	4–00	5–20	6–40	8–00	8–53.5	10–40	4.67	200
450	3–22.5	4–30	5–03.75	6–45	8–26.25	10–07.5	11–15	13–30	6.65	225
500	4–10	5–33.5	6.15	8–20	10–25	12–30	13–53.5	16–40	9.04	250

40 FEET.

per chord.

40′	45′		1° 00′		1° 15′		1° 30′		1° 40′		2° 00′	
p and *q*.												
q	*p*	*q*	*p*	*q*	*p*	*q*	*p*	*q*	*p*	*q*	*p*	*q*
40	0.07	40	0.09	40	0.12	40	0.14	40	0.16	40	0.19	40
60	0.24	60	0.31	60	0.39	60	0.47	60	0.52	60	0.63	60
80	0.56	80	0.75	80	0.93	80	1.12	80	1.24	80	1.49	80
100	1.08	100	1.45	100	1.81	100	2.18	100	2.42	100	2.90	100
120	1.89	120	2.51	120	3.15	120	3.78	119.9	4.19	119.9	5.02	119.9
140	2.98	140	3.98	139.9	4.97	139.9	5.95	139.9	6.61	139.8	7.95	139.8
160	4.46	159.9	5.95	159.9	7.42	159.8	8.90	159.8	9.90	159.7	11.90	159.6
179.9	6.38	179.9	8.50	179.8	10.60	179.7	12.70	179.6	14.10	179.4	16.90	179.2
199.8	8.70	199.8	11.60	199.7	14.50	199.5	17.40	199.3	19.30	199.0	23.10	198.7

50 FEET.

per chord.

40′		45′		1° 00′		1° 15′		1° 30′		1° 40′		2° 00′	
p and *q*.													
p	*q*	*p*	*q*	*p*	*q*	*p*	*q*	*p*	*q*	*p*	*q*	*p*	*q*
0.10	50	0.11	50	0.15	50	0.18	50	0.22	50	0.24	50	0.29	50
0.33	75	0.37	75	0.49	75	0.62	75	0.74	75	0.82	75	0.98	75
0.78	100	0.87	100	1.15	100	1.42	100	1.74	100	1.94	100	2.32	100
1.51	125	1.70	125	2.26	125	2.84	125	3.40	125	3.79	125	4.53	125
2.63	150	2.95	150	3.92	150	4.90	150	5.91	150	6.54	150	7.88	144.9
4.15	175	4.68	175	6.22	175	7.80	175	9.35	174.9	10.39	174.9	12.41	174.9
6.22	200	6.98	200	9.34	199.9	11.61	199.9	13.98	199.9	15.42	199.8	18.43	199.8
8.87	225	10.0	224.9	13.35	224.9	16.6	224.8	20.0	224.9	22.1	224.7	26.5	224.6
12.1	249.9	13.6	249.9	18.1	249.8	22.6	249.7	27.2	249.6	30.2	249.5	36.2	249.3

Deflection for station 190 + 87.9 is found as follows:

Deflection for circular arc for 30′ chord $= 30 \times 0.3 \times 4 = 36$ minutes.

Deflection for spiral for 30′ chord from P.S. = 0° 02′.

Deflection from tangent through station 190 + 57.9 (P.C.S.) = 0° 36′ − 0° 02′ = 0° 34′.

The same method, the theory of which is developed in second part of paragraph 38, is used to obtain the other deflections from P.C.S. to P.T.

The deflections from P.S. to P.S.C. are from the tangent thru the P.S. The deflections from P.S.C. to P.C.S. are from the tangent thru the P.S.C. The deflections from P.C.S. to P.T. are from the tangent thru the P.C.S.

46. The following problem gives the method for the solution of easement-curve problems by the use of the slide rule.

Problem 33. Find the deflections for a 3° 40′ curve with spirals at each end to connect the tangents shown, using a rate of gain of elevation of the outer rail of 2″ per second and a speed of 50 miles per hour.

$$l_c = \frac{S^3D}{2000} = 230 \text{ feet.}$$

Setting. Opposite 50 on the *D* scale set 2000 on the *B* scale, place the runner over 3.67 on the *B* scale, bring the right index of the *B* scale to the runner and opposite 50 on the *B* scale read 230 on the *A* scale, the decimal point for the result being found by rough figuring.

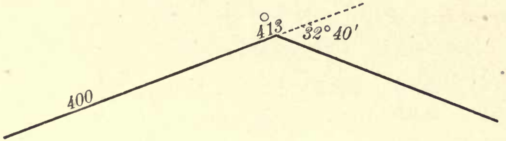


Fig. 38

Practically the same result may be found more quickly by the use of the diagram on page 116.

A six-chord curve of 40-foot chords will give a spiral 240 feet long. This length will be used, as it is in accordance with ordinary practice, but it will be shown that the 230-foot spiral can be almost as easily solved.

Let P.S. be the beginning of the spiral at the tangent end,

P.S.C. be the ending of the spiral at the circular curve end,

P.C.S. be the beginning of the ending spiral at the circular curve end,

P.T. be the end of the ending spiral,
n be the number of chords in the spiral,
l be the chord length in feet,
i_1 be the deflection from the tangent at the P.S. to the end of the first chord.

Then

$$i_1 = 0.1 \times l \frac{D}{n} = 0.1 \times 40 \frac{3.67}{6} = 2.44 \text{ minutes.}$$

Setting. Opposite 4 on the D scale set 6 on the C scale and opposite 3.67 on the C scale read 2.44 on the D scale.

For the station of the P.S. subtract from the station of the vertex the sum of the tangent distance of the circular curve of central angle of 32° 40′, the half length of the spiral and the spiral offset multiplied by the tan 16° 20′, or

$$T_s = T_c + q + p \tan \frac{I}{2}.$$

$$T_c = \frac{3438 \times 100}{220} \tan 16° 20' = \qquad 457 \text{ feet.}$$

$$q = \qquad 120 \text{ feet.}$$

$$p = \frac{240^2}{24 R_c} = \frac{240^2}{24 \times 1560} = 1.54 \text{ feet.}$$

$$1.54 \tan 16° 20' = \qquad 0.5 \text{ feet.}$$

$$T_s = \qquad 577.5 \text{ feet.}$$

V is at station	413
	$5 + 77.5$
P.S. is at station	$407 + 22.5$
l_c =	$2 + 40$
P.S.C. is at station	$409 + 62.5$

The deflections for the spiral vary as the squares of the distances from the P.S. Hence the deflection for the end of the spiral is

$$i_6 = 6^2 i_1 = 6^2 \times 2.44 = 87.75' = 1^\circ\, 27.75'$$
$$s_6 = 3\, i_6 = 4^\circ\, 23'.$$

As there are two spirals the total amount of central angle used up by the spirals is $2 \times 4^\circ\, 23' = 8^\circ\, 46'$, leaving $32^\circ\, 40' - 8^\circ\, 46' = 23^\circ\, 54'$ in the central angle of the circular part of the curve.

Let l_c = length of the circular part

$$= \frac{23^\circ\, 54'}{3^\circ\, 40'} \text{ in stations}$$

$$= \frac{1434}{220} = 6.52 \text{ stations.}$$

P.S.C.	$409 + 62.5$
L_c	$6 + 52$
P.C.S.	$416 + 14.5$
l_c	$2 + 40$
P.T.	$418 + 54.5$

Station.	Deflection.	Station.	Deflection.
407+22.5	P.S.	409+62.5	P.S.C.
+62.5	0° 02.44′	410	0° 41′
408	0° 09.2′	411	2° 31′
+02.5	0° 09.8′	412	4° 21′
+42.5	0° 22′	413	6° 11′
+82.5	0° 39′	414	8° 01′
409	0° 48′	415	9° 51′
+22.5	1° 01′	416	11° 41′
+62.5	1° 28′	+14.5	11° 57′

Station.	Deflection.
416+14.5	P.C.S.
+54.5	0° 41.6′
+94.5	1° 18.2′
417	1° 24′
+34.5	1° 52′
+74.5	2° 17′
418	2° 31.5′
+14.5	2° 39′
+54.5 P.T.	2° 56′

Number of chords from

$$\text{Sta. } 407 + 22.5 \text{ to Sta. } 408 = \frac{77.5}{40} = 1.94,$$

$$\text{Sta. } 407 + 22.5 \text{ to Sta. } 409 = \frac{177.5}{40} = 4.44.$$

The deflections for the first spiral are found from the following setting: Opposite 2.44 on

the A scale set the index of the slide and opposite 1.94, 2, 3, 4, 4.44, 5 and 6 on the C scale read 9.2, 9.8, 22, 39, etc., on the A scale.

The setting for the deflections of the circular part has been given in a preceding problem.

The deflections for the circular part are from the tangent thru the P.S.C.

The deflections for the ending spiral are from the tangent thru the P.C.S. and are found by subtracting the deflection for the spiral for the given chord length from the deflection for the circular curve and for the same chord length, e.g., the deflection for 40 feet for a 3° 40′ curve is 44′ and for 40 feet for the spiral is 2.44′, as found above; then the deflection for the end of the first 40-foot chord of the ending spiral is 44′ − 2.4′ = 41.6′.

The deflections for the ending spiral are found as follows:

Station.	Circular curve.	Spiral.	Ending spiral.
416+14.5	....	....	P.C.S.
+54.5	0° 44′	0° 02.4′	0° 41.6′
+94.5	1° 28′	0° 09.8′	1° 18.2′
417	1° 34′	0° 10′	1° 24′
+34.5	2° 14′	0° 22′	1° 52′
+74.5	2° 56′	0° 39′	2° 17′
418	3° 24′	0° 52.5′	2° 31.5′
+14.5	3° 40′	1° 01′	2° 39′
+54.5	4° 24′	1° 28′	2° 56′

After the stations of the P.S., P.S.C., P.C.S. and P.T. have been determined, the skeleton for the rest of the computation can be laid out and all the deflections for the circular part, including those for finding the deflections of the ending spiral, can be found from one setting of the slide rule. Likewise all the deflections for the spirals can be found from one setting of the slide rule.

If spirals, each 230 feet long consisting of five 40-foot chords and one 30-foot chord, are used in place of the above, the following solution is made:

$$T_s = T_c + q + p \tan \frac{I}{2}.$$

$$T_s = 457 + 115 + \frac{230^2}{24 R_c} \tan 16° \, 20'$$

$$= 572.4 \text{ feet.}$$

V at	413
$T_s =$	5 + 72.4
P.S. at	407 + 27.6
$l_c =$	2 + 30
P.S.C. at	409 + 57.6
$L_c =$	6 + 61
P.C.S. at	416 + 18.6
$l_c =$	2 + 30
P.T. at	418 + 48.6

$$S_c = \frac{115}{100} \times 3° \, 40' = 4° \, 13'.$$

$$i_c = \frac{S_c}{3} = 1° \, 24'.$$

$$i_1 = \frac{40^2}{230^2} \times 1° \, 24' = 2.52'.$$

Number of 40-foot chords from

$$407 + 27.6 \text{ to } 408 = \frac{72.4}{40} = 1.81.$$

$$407 + 27.6 \text{ to } 409 = \frac{172.4}{40} = 4.31.$$

$$407 + 27.6 \text{ to } 409 + 57.6 = \frac{230}{40} = 5.75.$$

$$2\,S_c = 8° \, 26'.$$

$$I_c = 32° \, 40' - 8° \, 26' = 24° \, 14'.$$

$$L_c = \frac{24° \, 14'}{3° \, 40'} = \frac{1454}{220} = 6.61 \text{ stations.}$$

Station.	Deflection.	Station.	Deflection.
407+27.6	P.S.	409+57.6	P.S.C.
+67.6	0° 02.52′	410	0° 46.5′
408	0° 08.3′	411	2° 36.5′
+07.6	0° 10.4′	412	4° 26.5′
+47.6	0° 22.6′	413	6° 16.5′
+87.6	0° 40.5′	414	8° 06.5′
409	0° 47′	415	9° 56.5′
+27.6	1° 03′	416	11° 46.5′
+57.6	1° 24′	+18.6	12° 07′

Station.	Deflection.	Circular curve.	Spiral.
416+18.6	P.C.S.	....	....
+58.6	0° 41.5′	0° 44′	0° 02.5′
+98.6	1° 17.5′	1° 28′	0° 10.4′
417	1° 19′	1° 29.5′	0° 10.5′
+38.6	1° 51.4′	2° 14′	0° 22.6′
+78.6	2° 15.5′	2° 56′	0° 40.5′
418	2° 27′	3° 19′	0° 52′
+18.6	2° 37′	3° 40′	1° 03′
+48.6	2° 48′	4° 12′	1° 24′

This solution shows the flexibility of this method which may be used for any length of spiral with any unit chord length. With a little experience with the slide rule solutions can be made more rapidly than by the use of tables.

47. The following gives in a concise form the equation for finding the deflections for a spiral and for which the use of the slide rule is particularly adapted.

By Equation (36),

$$i = lNC \times 0.1$$

Let $L =$ the chord length.

Then $NL = l$

and $$i = N^2 \frac{LC}{10}. \quad \ldots \quad (41)$$

The next table gives the deflections, for a change of curvature of one degree per chord, at the end of chords of 10 feet each.

Deflections for chords of 10 feet and for a change of curvature of one degree per chord.

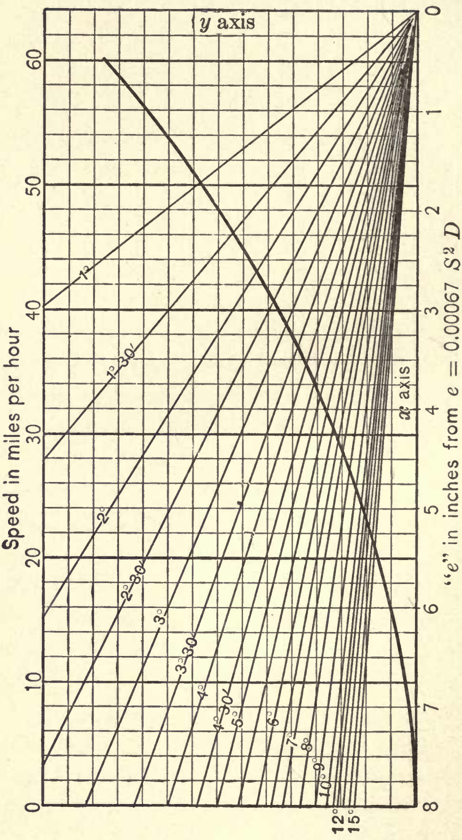
1	2	3	4	5	6	7	8	9	10
0° 01′	0° 04′	0° 09′	0° 16′	0° 25′	0° 36′	0° 49′	1° 04′	1° 21′	1° 40′

For any other chord length and change of curvature per chord, multiply the tabular amount by the given change of curvature, in degrees, per chord, and then multiply this product by the given chord length divided by 10, the result being the required deflection.

For fractional chords multiply the deflection, found for the first chord, by the squares of the fractions which represent the distances of the points from the beginning of the spiral, expressed in chords.

48. The following diagram is for finding e in inches from the speed of the train and the degree of the curve.

In using it find the intersection of the curve and the line of the adopted speed, then go parallel to the x axis to the intersection with the line of the degree of the curve, then go


y axis
Speed in miles per hour
0
10
20
30
40
50
60
x axis
"e" in inches from e = 0.00067 S² D
0
1
2
3
4
5
6
7
8
1°
1° 30′
2°
2° 30′
3°
3° 30′
4°
4° 30′
5°
6°
7°
8°
9°
10°
12°
15°

parallel to the y axis and read the result on the bottom line of the diagram, e.g., to find the value of e for a speed of 40 miles per hour and for a 4° curve, the result would be 4.6 inches.

49. The next diagram is for finding l_c in feet from the adopted speed in miles per hour, the degree of the curve and the adopted rate of gaining the elevation of the outer rail.

In using it, find the intersection of the line of the adopted speed and the curve, then go parallel to the x axis to the intersection with the line of the degree of the curve, then go parallel to the y axis to the intersection with the line of the adopted rate of gain of the elevation of the outer rail, then go parallel to the x axis to the left line of the diagram and read the length of the spiral, e.g., to find the length of the spiral for a 4° curve, speed 40 miles per hour and a rate of gaining the elevation of the outer rail of 1.6 inches per second, find the intersection of the 40 miles per hour line with the curve, then go parallel to the x axis to the 4° line, then go parallel to the y axis to the 1.6 line, then go parallel to the x axis to the left line of the diagram and read 160 feet.

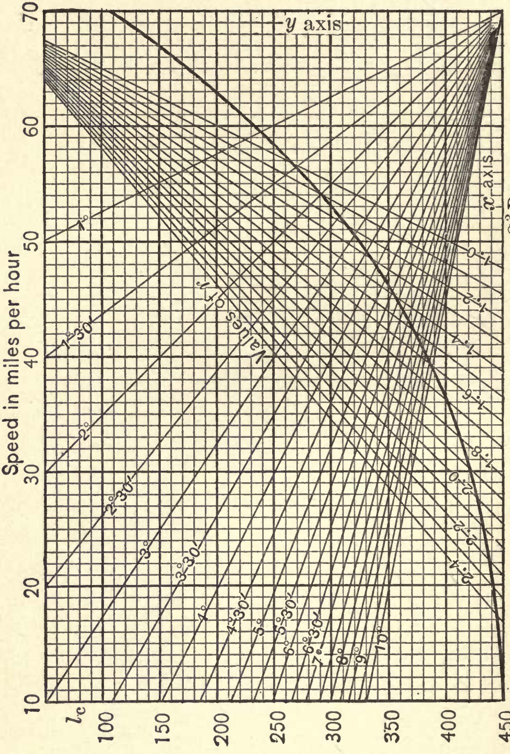


Diagram for l_c from $l_c = \dfrac{S^3 D_c}{1000\,r}$

CHAPTER VII

Earthwork

50. For finding the volumes of earthwork in railroad grading, it is necessary to take cross-sections in the field and from the notes taken the volumes may be figured.

The method of taking cross-sections where an ordinary level is used is as follows:

1st. From the grade line established on the profile made from the levels run over the center line of the survey, find the elevation of grade at the station where the cross-section is to be taken.

2nd. From the readings taken on some bench mark and on turning points, find the H.I. of the level.

3rd. Subtract the elevation of grade from the H.I. of the level and the result is r_g, the rod to grade.

4th. Read the rod held at the center line stake and subtract this reading, r_s, from r_g and the result is the center cut or fill, cut if $+$ and fill if $-$.

5th. Assuming the section to be a level

section, figure, from the given width of road-bed and ratio of the side slope, the distance from the center line to the edge of the side slope (equal to one-half of the width of the road-bed plus the side slope times the center cut or fill). Going out this distance estimate the difference of elevation between this point and the center line point. Apply this difference to the cut or fill at the center and from the resulting amount of cut or fill figure anew the distance to the edge of the side slope. Go out this distance from the center stake and on a line at right angles to the center line. At the point thus found take a rod reading. Subtract this reading from the r_g and the result is the cut or fill. From this cut or fill figure anew the distance out from the center and if this exceeds the actual distance out, go out further and if less than the actual distance go nearer the center line, until a point is found where the reading of the rod gives a cut or fill that corresponds with the distance from the center line. Where the section is not level across the top, the three-level form of section is usually found.

51. The following is a form for the notes of cross-sections:

Station.	Elevation.	Grade.	L	C	R
85	105.3	105.2	0.0/10.0	+0.1	+0.4/10.6
84	107.4	104.6	+2.0/13.0	+2.8	+3.2/14.8
83	110.2	104.0	+4.2/16.3	+6.2	+7.0/20.5
82	109.4	103.4	+4.2/16.3	+6.0	+7.2/20.8
81	106.6	102.8	+3.0/14.5	+3.8	+4.0/16.0
80	107.8	102.2	+4.6/16.9	+5.6	+5.8/18.7

52. Where the grade contour does not cross the center line at right angles at a point where there is a change from cut to fill or vice versa, Fig. 39 shows the points that must be taken in cross-section work.

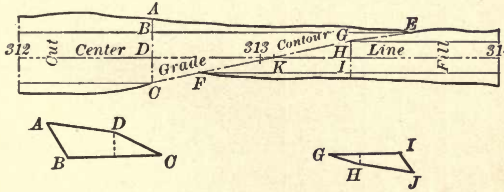


Fig. 39

D is at station 312 + 60, *F* is opposite station 312 + 78, *K* is at station 313 + 06, *H* is at station 313 + 38, and *E* is opposite station 313 + 62. The lower diagrams show the forms of the cross-sections at *D* and *H* respectively.

From station 312 to D is in cut and may be figured by the average end area method. From D to E is in cut and may be figured as a pyramid of the base $ABCD$ and of the height equal to the distance from D to E measured along the center line. From F to H is in fill and may be figured as a pyramid of the base $GHJI$ and of the height equal to the distance from F to H measured along the center line. From H to station 314 is in fill and may be figured by the average area method.

The following gives the notes for this part of the cross-section work:

Station.	Elevation.	Grade.	L	C	R
314	164.0	166.1	−1.8/9.7	−2.1	−2.4/10.6
313+62	....	....	0.0/10.0	......	....
313+38	165.7	166.7	0.0/7.0	−1.0	−1.2/8.8
313+06	167.0	167.0	....	0.0	....
313	167.4	167.1	....	+0.3	....
312+78	....	....	....	......	0.0/7.0
312+60	170.2	167.5	+3.4/15.1	+2.7	0.0/10.0
312	172.4	168.1	+5.2/17.8	+4.3	+3.2/14.8

53. A level section is one at which the surface of the ground at right angles to the center line is horizontal. The area of a level section is found as follows:

Let b = the width of the roadbed in feet.
s = the ratio of the side slope, horizontal to vertical.
c = the center cut or fill in feet.

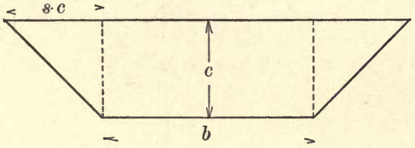


Fig. 40

From Fig. 40, it is seen that the area of the section is equal to $bc + sc \times c$,

or $$A = (b + sc)c. \quad . \quad . \quad . \qquad (42)$$

54. A three-level section is shown in Fig. 41. The amounts of the cut or fill at the center and the edges of the side slopes must be found. The area may be expressed in two ways:

1st. By the use of the grade triangle. In Fig. 41, *ABFDE* is the three-level section and *BHF* is its grade triangle.

$$GH = \frac{BG}{s} = \frac{b}{2s}.$$

The figure *AHDE* may be divided into two triangles with the common base $EH = c + \dfrac{b}{2s}$ and of altitudes d_l and d_r respectively.

$$\text{Area } ABFDE = \left(c + \frac{b}{2\,s}\right)\frac{d_r + d_l}{2} - \frac{b}{2\,s} \times \frac{b}{2}.$$

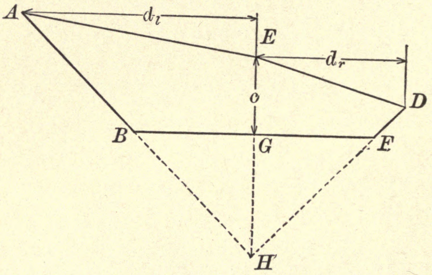


Fig. 41

Let $$D = d_r + d_l.$$

Then

$$ABFDE = \left(c + \frac{b}{2\,s}\right)\frac{D}{2} - \frac{b^2}{4\,s}. \quad . \quad (43)$$

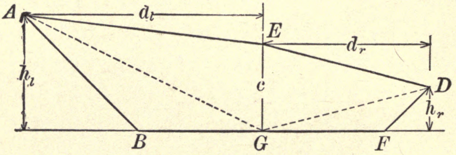


Fig. 42

2nd. Without the use of the grade triangle. In Fig. 42, the area *ABFDE* is divided into four triangles, *ABG* and *GFD* having equal

bases, each equal to $\frac{b}{2}$, and AGE and GED having the same altitude c.

$$\text{Area } ABFDE = \frac{c}{2}(d_r + d_l) + \frac{b}{2}\frac{h_r + h_l}{2}. \quad (44)$$

55. The two methods most commonly used for figuring volumes of earthwork are:

1st. The average end area, which is obtained by simply dividing the sum of the end areas by two and multiplying the result by the distance between the end sections. The last result divided by 27 gives the volume in cubic yards, if the measurements given in the notes are in feet.

2nd. The use of the prismoidal formula for precise results. If A_1 and A_2 are the end areas, A_m the middle area and h the length of the earthwork section, then, by the prismoidal formula,

$$V = \frac{h}{6}(A_1 + 4\,A_m + A_2). \quad . \quad . \quad (45)$$

A_m may be found by taking a mean of the dimensions of the end sections and figuring its area from these as its dimensions.

56. By Equation (46) the prismoidal correction may be found. This correction sub-

tracted from the volume given by the average end area method gives the precise result.

$$C_P = \frac{1}{3.24}(c_1 - c_0)(D_1 - D_0). \quad . \quad (46)$$

C_P is in cubic yards if c_1, c_0, D_1 and D_0 are in feet.

In Fig. 43, the volume $A_1G_1F_1F_0G_0A_0$ is the same, whether figured by the average end

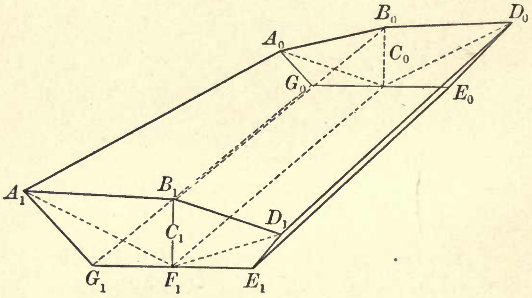


Fig. 43

area method or by the prismoidal formula, because $G_1F_1 = G_0F_0$. Likewise, the volume $F_1D_1E_1E_0F_0D_0$ needs no correction if figured by the average end area method.

If the entire volume of earthwork between sections zero and 1 is figured by the average end area method, then the correction to be applied,

to get precise results, comes from the parts $A_1B_1F_1F_0A_0B_0$ and $B_1D_1F_1F_0B_0D_0$. For the sum of these parts the area at section 1 $= \dfrac{c_1D_1}{2}$, at section zero $= \dfrac{c_0D_0}{2}$, and at the middle section $= \dfrac{c_0 + c_1}{2}\,\dfrac{D_0 + D_1}{2 \times 2}$, where c_0 and c_1 are the respective center heights and D_0 and D_1 are the respective total widths of the sections. Substituting these values in the equations for the volumes,

$$V_E = \frac{\dfrac{c_1D_1}{2} + \dfrac{c_0D_0}{2}}{2}\,l = \frac{l}{4}(c_1D_1 + c_0D_0).$$

$$V_E = \frac{l}{12}(3\,c_1D_1 + 3\,c_0D_0).$$

$$V_P = \frac{l}{6}\left(\frac{c_1D_1}{2} + 4\,\frac{c_0 + c_1}{2}\,\frac{D_0 + D_1}{2 \times 2} + \frac{c_0D_0}{2}\right)$$

$$= \frac{l}{12}(2\,c_1D_1 + c_0D_1 + c_1D_0 + 2\,c_0D_0).$$

Subtract V_P from V_E for C_P.

$$C_P = \frac{l}{12}(c_1D_1 - c_1D_0 - c_0D_1 + c_0D_0)$$

$$= \frac{l}{12}(c_1 - c_0)(D_1 - D_0).$$

This expression shows that if the similar dimensions of the end section have about the same values, the average end area method gives close results.

If $l = 100$ feet and C_P is expressed in cubic yards

$$C_P = \frac{100}{12 \times 27}(c_1 - c_0)(D_1 - D_0)$$

$$= \frac{1}{3.24}(c_1 - c_0)(D_1 - D_0.)$$

57. For finding the amounts of earthwork from cross-section notes, the slide rule does not give as rapid solutions as diagrams or tables. Where these are not available its use is advisable. For this class of problems the following equations are in forms readily solved by the slide rule.

1st. For volumes, in cubic yards, of level sections for lengths of 50 feet,

$$V_{50} = \frac{50}{27}cb + \frac{50}{27}sc^2.$$

Where c is the center cut or fill, b is the width of roadbed and s is the ratio of the side slope, horizontal to vertical.

For $b = 20$ feet and $s = 1\frac{1}{2}$,

$$V_{50} = 37.04\ c + 2.78\ c^2;$$

For $b = 14$ feet and $s = 1\frac{1}{2}$,

$$V_{50} = 25.95\,c + 2.78\,c^2;$$

For $b = 33$ feet and $s = 1\frac{1}{2}$,

$$V_{50} = 61.11\,c + 2.78\,c^2;$$

For $b = 27$ feet and $s = 1\frac{1}{2}$,

$$V_{50} = 50\,c + 2.78\,c^2.$$

2nd. For volumes, in cubic yards, of three level sections for lengths of 50 feet,

$$A = \frac{D}{2}\left(c + \frac{b}{2\,s}\right) - \frac{b^2}{4\,s},$$

$$V_{50} = \frac{50}{54}Dc + \frac{50}{54}\,\frac{b}{2\,s}\,(D - b).$$

Where A is the area of the section in square feet and D is the entire width of the section, from left side height to right side height, and the other letters represent the same quantities as in the first case.

For $b = 20$ feet and $s = 1\frac{1}{2}$,

$$V_{50} = \frac{100}{108}Dc + 6.17\,(D - b);$$

For $b = 14$ feet and $s = 1\frac{1}{2}$,

$$V_{50} = \frac{100}{108}Dc + 4.32\,(D - b);$$

For $b = 33$ feet and $s = 1\frac{1}{2}$,

$$V_{50} = \frac{100}{108} Dc + 10.18\,(D - b);$$

For $b = 27$ feet and $s = 1\frac{1}{2}$,

$$V_{50} = \frac{100}{108} Dc + \ 8.33\,(D - b).$$

The following problems show the use of the slide rule for solving earthwork problems.

Problem 34. Find the volume, in cubic yards, of earthwork from station 116 to station 118, assuming that the sections are level sections.

$$b = 14 \text{ feet.} \qquad s = 1\tfrac{1}{2}.$$

$$V_{50} = 25.95\,c + 2.78\,c^2.$$

Stations.	Elevations.	Grade.	c
316	315.1	318.2	− 3.1
317	313.6	319.0	− 5.4
318	316.8	319.8	− 3.0
316	80.5	+ 26.6	=107.1
317	(140.1	+ 81.0)×2	=442.2
318	77.9	+ 25.0	=102.9
			652.2 cu. yds.

Problem 35. Find the volume, in cubic yards, of the earthwork from station 212 to

station 214, assuming that the sections are level sections.

$$b = 33 \text{ feet.} \qquad s = 1\tfrac{1}{2}.$$

$$V_{50} = 61.11\,c + 2.78\,c^2.$$

Stations.	Elevations.	Grade.	c
212	153.9	147.2	+ 6.7
213	150.4	146.6	+ 3.8
214	146.9	146.0	+ 0.9
212	409.0	+124.5	=533.5
213	2(232.0	+ 40.0)	=544.0
214	54.9	+ 2.2	= 57.1
			1134.6 cu. yds.

Problem 36. Find the volume, in cubic yards, of the earthwork from station 272 to station 274.

$$b = 20 \text{ feet.} \qquad s = 1\tfrac{1}{2}.$$

$$V_{50} = \tfrac{100}{108}\,Dc + 6.17\,(D - b).$$

Stations.	Elevations.	Grade.	L	C	R
272	153.9	147.2	$\frac{+7.8}{21.7}$	+6.7	$\frac{+6.0}{19.0}$
273	150.4	146.6	$\frac{+5.4}{18.1}$	+3.8	$\frac{+3.2}{14.8}$
274	146.9	146.0	$\frac{+2.2}{13.3}$	+0.9	$\frac{+0.6}{10.9}$
272	252.5 +127.5 =380.0				
273	2(115.5 + 79.5) =390.0				
274	20.1 + 25.9 = 46.0				
	816.0 cu. yds.				

Problem 37. Find the volume, in cubic yards, of earthwork from station 376 to station 378.

$$b = 27 \text{ feet.} \qquad s = 1\tfrac{1}{2}.$$

$$V_{50} = \frac{100}{108} Dc + 8.33\,(D - b).$$

Stations.	Elevations.	Grade.	L	C	R
376	315.1	318.2	$\frac{-1.2}{15.3}$	-3.1	$\frac{-4.6}{20.4}$
377	313.6	319.0	$\frac{-2.8}{17.7}$	-5.4	$\frac{-7.8}{25.2}$
378	316.8	319.8	$\frac{-2.2}{16.8}$	-3.0	$\frac{-3.4}{18.6}$
376	102.5	+ 72.5	=175.0	......	......
377	2(214.5	+132.5)	=694.0	......	......
378	98.3	+ 70.0	=168.3	......	......
			1037.3 cu. yds.		

Problem 38. Find the prismoidal correction for the volume found in Problem 36.

$$C_P = \frac{1}{3.24}(c_1 - c_0)(D_1 - D_0).$$

From station 272 to station 273

$$C_P = \frac{2.9 \times 7.8}{3.24} = 6.98.$$

From station 273 to station 274

$$C_P = \frac{2.9 \times 8.7}{3.24} = \frac{7.78}{14.76} \text{ cu. yds.}$$

Problem 39. Find the prismoidal correction for the volume found in Problem 37.

From station 376 to station 377

$$C_P = \frac{2.3 \times 7.2}{3.24} = 5.11.$$

From station 377 to station 378

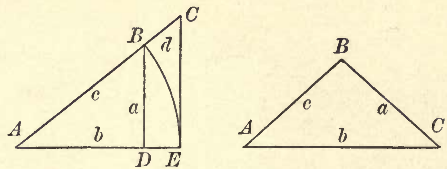
$$C_P = \frac{2.4 \times 7.5}{3.24} = \frac{5.55}{10.66} \text{ cu. yds.}$$

There are other problems in curves and earthwork in which the slide rule may be used to advantage. However, a sufficient number of problems has been given to guide the student in deciding when to use it.

CONSTANTS

Degrees in arc = radius	57.296°
Minutes in arc = radius	3437.75′
Seconds in arc = radius	206264.8″
Ratio of circumference to diameter	$3.14159 = \pi$
sin 1′	0.00029
sin 1°	0.01745
1 English inch	0.0254 meters
1 English foot	0.3048 meters
1 English statute mile	1.6093 kilometers
1 meter	39.37 inches
1 meter	3.2808 feet
1 kilometer	0.6214 miles
1 square meter	10.7639 sq. ft.
1 kilogram	2.2046 av. lbs.
Length of seconds pendulum at N. Y.	39.1017 inches
Acceleration due to gravity at N. Y.	32.1595 feet
Cubic inches in 1 U. S. gallon	231
Cubic inches in 1 Imperial gallon	277.274
U. S. gallons in 1 cubic foot	7.4805
Feet in 1 mile	5,280
Square feet in 1 acre	43,560
Area of a circle	$\frac{\pi d^2}{4}$ or πr^2
Surface of a sphere	πd^2
Volume of a sphere	$\frac{4}{3}\pi r^3$
Area of sector of circle (arc=l)	$\frac{1}{2} lr$
Area of segment of parabola (c=chord and m=mid. ord.)	$\frac{2}{3} cm$
Area of segment of circle (approx.)	$\frac{2}{3} cm$
Difference between the base and hypothenuse of right triangle.	$\frac{a^2}{2b}$ or $\frac{a^2}{2h}$
Area of an ellipse of semi-axes a and b.	πab

FORMULAS



Simple Triangles.

1. $\sin A = \frac{a}{c} = \cos B.$

2. $\tan A = \frac{a}{b} = \cot B.$

3. $\sec A = \frac{c}{b} = \operatorname{cosec} B.$

4. $\operatorname{vers} A = \frac{c-b}{c} = \frac{e}{c}.$

5. $\operatorname{exsec} A = \frac{d}{c}.$

6. $a = c \sin A = b \tan A = c \cos B = b \cot B = \sqrt{(c+b)(c-b)}.$

7. $b = c \cos A = a \cot A = c \sin B = a \tan B = \sqrt{(c+a)(c-a)}.$

8. $c = \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{a}{\cos B} = \frac{b}{\cos A} = \frac{e}{\operatorname{vers} A} = \frac{d}{\operatorname{exsec} A}.$

FORMULAS

9. $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ in triangle ABC.

10. $a = b\cos C + c\cos B$.

11. $a^2 = b^2 + c^2 - 2\,bc\cos A$.

12. $\dfrac{a+b}{a-b} = \dfrac{\tan\frac{1}{2}(A+B)}{\tan\frac{1}{2}(A-B)}$.

13. If $s = \frac{1}{2}(a+b+c)$,

$$\sin\frac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{bc}},$$

$$\cos\frac{1}{2}A = \sqrt{\frac{s(s-a)}{bc}},$$

$$\tan\frac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}.$$

14. Area $ABC = \frac{1}{2}ab\sin C$

$$= \frac{c^2}{2}\,\frac{\sin A\sin B}{\sin C}$$

$$= \sqrt{s(s-a)(s-b)(s-c)}.$$

15. $\text{vers}\,A = \dfrac{2(s-b)(s-c)}{bc}$.

Trigonometric

16. $\sin A = \sqrt{1-\cos^2 A} = \tan A\cos A = 2\sin\frac{1}{2}A\cos\frac{1}{2}A$.

17. $\cos A = \sqrt{1-\sin^2 A} = \cos^2\frac{1}{2}A - \sin^2\frac{1}{2}A$.

18. $\tan A = \dfrac{\sin A}{\cos A} = \dfrac{\sin 2A}{1+\cos 2A} = \dfrac{2\tan\frac{1}{2}A}{1-\tan^2\frac{1}{2}A}$.

19. $\cot A = \dfrac{\cos A}{\sin A} = \dfrac{\sin 2A}{1-\cos 2A} = \dfrac{\cot^2\frac{1}{2}A - 1}{2\cot\frac{1}{2}A}$.

20. $\text{vers}\,A = 1-\cos A = \sin A\tan\frac{1}{2}A = 2\sin^2\frac{1}{2}A$.

21. $\text{exsec}\,A = \sec A - 1 = \tan A\tan\frac{1}{2}A = \dfrac{1-\cos A}{\cos A}$.

22. $\sin 2A = 2\sin A\cos A$.

23. $\sin(A\pm B) = \sin A\cos B \pm \cos A\sin B$.

24. $\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A$.

25. $\cos(A\pm B) = \cos A\cos B \mp \sin A\sin B$.

26. $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$.

27. $\tan(A\pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$.

28. $\cot 2A = \dfrac{\cot^2 A - 1}{2\cot A}$.

29. $\sin A + \sin B = 2\sin\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$.

30. $\sin A - \sin B = 2\cos\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$.

31. $\cos A + \cos B = 2\cos\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$.

32. $\cos A - \cos B = -2\sin(A+B)\sin\frac{1}{2}(A-B)$.

33. $\sin A\sin B = \frac{1}{2}\cos(A-B) - \frac{1}{2}\cos(A+B)$.

34. $\cos A\cos B = \frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B)$.

35. $\sin A\cos B = \frac{1}{2}\sin(A+B) + \frac{1}{2}\sin(A-B)$.

FORMULAS

36. $\sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A = \sin(A+B)\sin(A-B)$.

37. $\cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$
$= \cos(A+B)\cos(A-B)$.

38. $2\sin^2 \frac{1}{2} A = 1 - \cos A$.

39. $2\cos^2 \frac{1}{2} A = 1 + \cos A$.

40. $\tan \frac{1}{2} A = \pm\sqrt{\frac{1-\cos A}{1+\cos A}} = \frac{\sin A}{1+\cos A} = \frac{1-\cos A}{\sin A}$.

41. $\tan A + \tan B = \frac{\sin(A+B)}{\cos A \cos B}$.

42. $\tan A - \tan B = \frac{\sin(A-B)}{\cos A \cos B}$.

Trigonometric Series

(The angles are in arcs of unit radius)

43. $\sin x = \frac{x}{1} - \frac{x^3}{2\times 3} + \frac{x^5}{2\times 3\times 4\times 5}$
$- \frac{x^7}{2\times 3\times 4\times 5\times 6\times 7} + \dots$

44. $\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{2\times 3\times 4} - \frac{x^6}{2\times 3\times 4\times 5\times 6} + \dots$

45. $\tan x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \frac{62}{2835}x^9 + \dots$

46. $\cot x = \frac{1}{x} - \frac{1}{3}x - \frac{1}{45}x^3 - \frac{2}{945}x^5 - \frac{1}{4725}x^7 - \dots$

Simple Curves

47. $R = \frac{50}{\sin \frac{1}{2} D}$.

48. $R = \frac{100 \times 3438}{D'}$ (where D is in minutes).

49. $R = \frac{5730}{D^\circ}$ (ap.).

50. $R = \frac{5730}{D^\circ} + 0.07\,D^\circ$.

51. $T = R \tan \frac{1}{2} I = \frac{T_1}{D}$.

52. $E = R \operatorname{exsec} \frac{1}{2} I = R \tan \frac{1}{2} I \tan \frac{1}{4} I$.

53. $M = R \operatorname{vers} \frac{1}{2} I = R \sin \frac{1}{2} I \tan \frac{1}{4} I$.

54. $C = 2R \sin \frac{1}{2} I$.

55. $c = 2R \sin \frac{1}{2} d$.

56. $d = \frac{cD}{100}$ (ap.).

57. $\frac{d}{2} = c \times 0.3 \times D^\circ$ (result in minutes).

58. $L = \frac{I}{D}$ (ap.) $= 100\,\frac{I}{D}$ (in feet).

59. $a = \frac{c^2}{2R}$. $a_{100} = \frac{100^2}{2R}$.

60. $a_n = \frac{7}{8} n^2 D^\circ$ (ap.). (n is in sta's.)

61. a_n (between curves) $= \frac{7}{8} n^2 (Df - Di)$ (ap.).

FORMULAS

62. $M = \dfrac{c^2}{8R}$.

63. $\text{Ordinate} = \dfrac{\left(\frac{c}{2} + b\right)\left(\frac{c}{2} - b\right)}{2R}$.

64. $AA' = \dfrac{p}{\sin I}$.

65. $R - R' = \dfrac{p}{\text{vers } I} = \dfrac{p}{\sin I \tan \frac{1}{2} I} = \dfrac{p}{2 \sin^2 \frac{1}{2} I}$.

66. $R - R' = \dfrac{p}{\text{exsec } I} = \dfrac{p}{\tan I \tan \frac{1}{2} I}$.

67. $R = \dfrac{d^2}{4p}$.

68. $d = 2\sqrt{Rp}$.

69. $R = \dfrac{l}{\tan \frac{1}{2} I_B + \tan \frac{1}{2} I_C}$.

Vertical Curves

70. $y^2 = 2p'x$.

71. $a_1 = \dfrac{g - g'}{4n}$. $\left\{ \begin{array}{l} (n \text{ is No. of sta's in one-half of the} \\ \text{vert. curve.)} \end{array} \right.$

72. $a_2 = 4a_1$. $a_3 = 9a_1 \ldots$

Turnouts

73. $\sin S = \dfrac{t}{l}$.

74. $\cot \frac{1}{2} F = 2n$.

75. $R + \frac{1}{2} g = \dfrac{g - t}{\cos S - \cos F}$.

76. $L = l + \dfrac{g - t}{\tan \frac{1}{2}(F + S)}$.

77. $R + \frac{1}{2} g = \dfrac{g - t}{2 \sin \frac{1}{2}(F - S) \tan \frac{1}{2}(F + S)}$.

78. $\tan \frac{1}{2} O = \dfrac{gn}{Rm}$.

79. $R_t = \dfrac{gn}{\tan \frac{1}{2}(F \pm O)}$.

80. $R_s = 2gn^2$.

81. $L_t = 2R_t \sin \frac{1}{2}(F \pm O)$.

82. $L_s = 2gn$.

83. $D_t = D_s \pm D_m$.

84. $R_2 = (p - g)2n + \dfrac{p}{2}$.

85. $L_2 = \left(R_2 - \dfrac{g}{2}\right) \sin F$.

86. $L_2 = 2(p - g)n$.

FORMULAS

Easement Curves

87. $e = 0.00067\, S^2 D_c$ (inches).

88. $l_c = \dfrac{S^3 D_c}{1000\, r}$ (feet).

89. $Rl = R_c l_c$.

90. $\dfrac{D}{l} = \dfrac{D_c}{l_c}$.

91. $\dfrac{O_1}{l_1^3} = \dfrac{O_2}{l_2^3}$.

92. $s_E = \dfrac{l^2}{2\, R_c l_c}$.

93. $i_E = \dfrac{l^2}{6\, R_c l_c}$.

94. $i = \dfrac{s}{3}$.

95. $p = \dfrac{l_c^2}{24\, R_c}$.

96. $i = lNC \times 0.1$ (minutes).

97. $q = \dfrac{l_c}{2} - \dfrac{l_c^2}{48^2 R_c^2}$.

98. $T_s = q + R_c \tan \frac{1}{2} I + p \tan \frac{1}{2} I$.

99. $\dfrac{i_1}{l_1^2} = \dfrac{i_2}{l_2^2}$.

100. $i = N^2 \dfrac{LC}{10}$.

Earthwork

101. $A = (b + sc)\, c$ (level section).

102. $A = \left(c + \dfrac{b}{2\, s}\right) \dfrac{D}{2} - \dfrac{b^2}{4\, s}$ (3-level section).

103. $A = \dfrac{c}{2}\,(d_r + d_l) + \dfrac{b}{2}\,\dfrac{h_r + h_e}{2}$ (3-level section).

104. $V_P = \dfrac{h}{6}\,(A_1 + 4\, A_m + A_2)$ cu. ft.

105. $V_{AE} = h\, \dfrac{A_1 + A_2}{2}$ cu. ft.

106. $C_P = \dfrac{1}{3.24}\,(c_1 - c_0)\,(D_1 - D_0)$ cu. yds.

107. $V_{50} = 37.04\, c + 2.78\, c^2 \quad (b = 20', s = 1\frac{1}{2})$.

108. $V_{50} = 25.95\, c + 2.78\, c^2 \quad (b = 14', s = 1\frac{1}{2})$.

109. $V_{50} = 61.11\, c + 2.78\, c^2 \quad (b = 33', s = 1\frac{1}{2})$.

110. $V_{50} = 50\, c + 2.78\, c^2 \quad (b = 27', s = 1\frac{1}{2})$.

111. $V_{50} = \frac{100}{108}\, c + 6.17\,(D - b) \quad (b = 20', s = 1\frac{1}{2})$.

112. $V_{50} = \frac{100}{108}\, c + 4.32\,(D - b) \quad (b = 14', s = 1\frac{1}{2})$.

113. $V_{50} = \frac{100}{108}\, c + 10.18\,(D - b) \quad (b = 33', s = 1\frac{1}{2})$.

114. $V_{50} = \frac{100}{108}\, c + 8.33\,(D - b) \quad (b = 27, s = 1\frac{1}{2})$.